UNIT – 1

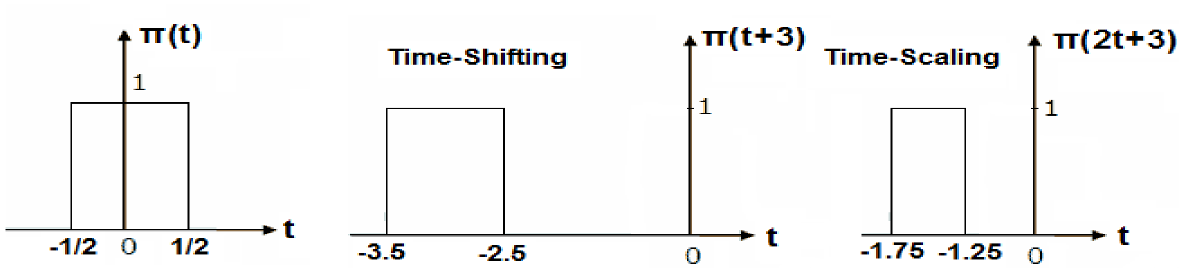
CLASSIFICATION OF SIGNALS AND SYSTEMS

PART A

1) State the properties of LTI system. **[Nov '11]**

Any LTI system is characterized by its impulse response & its behavior is described by its convolution operation. The properties of such systems are equivalent to the properties of convolution. Apart from being linear & time-invariant, the convolution is commutative, Associative & Distributive.

2) Draw the function π(2t+3) when **π(t) = 1 ; for t ≤ ½**
0 ; otherwise **[Nov '11]**



3) Determine whether the following signal is energy or power signal. And calculate its Energy or Power. x(t)=e^{-2t}u(t). **[Nov '12]**

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_0^\infty e^{-4t} dt = \frac{1}{4}$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_0^\infty e^{-4t} dt = 0$$

Since **Energy** is **finite** & **P**=**0**, x(t) is an **Energy** signal.

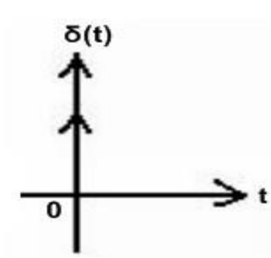
4) Check whether the following system is static or dynamic & also causal or non-causal. y(n)=x(2n). **[Nov '12]**

Since the output y(n) depends on the future input, **y(n)=x(2n)** is a **Dynamic** system & also a **Non-Causal** system.

5) Give the mathematical & graphical representation of CT & DT unit impulse function. **[Nov '13]**

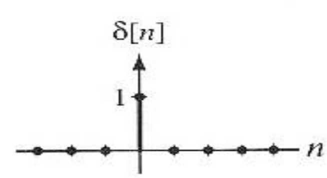
Unit Impulse (CT)

$$\int \delta(t)dt = 1; t = 0$$
$$= 0; t \neq 0$$



Unit Impulse (DT)

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



6) **What are the conditions for a system to be LTI system?** **[Nov '13]**

For an Linear **system** ; $T[ax_1(t)+b\,x_2(t)] = a\,y_1(t)+b\,y_2(t)$

For a **Time –Invariant system : y(t,T) = y(t-T)**

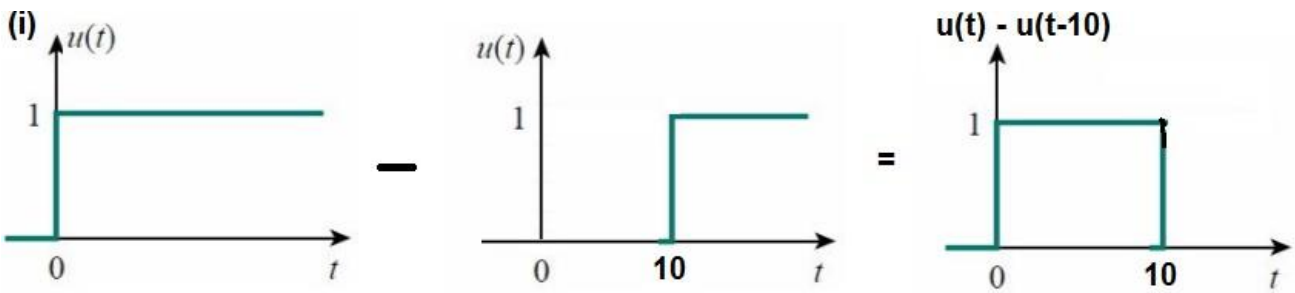
i.e., Response to a shifted input = Shifted or Delayed Response

7) **State any 2 properties of unit impulse function.** **[Nov '14](R13)**

(i) $\int x(t)\delta(t)dt = x(0)$

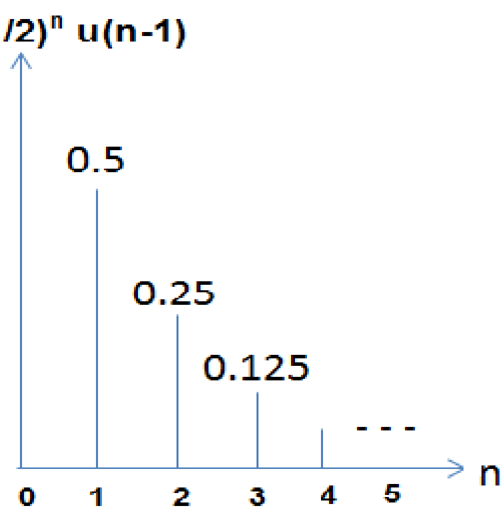
(ii) $\int x(t)\delta(t-t_o)dt = x(t_o)$

8) **Draw the following signals. (i) u(t) – u(t-10)(ii) $(1/2)^n$ u(n-1)** **[Nov '14](R13)**



(ii) Let x(n) = $(1/2)^n$ u(n-1)

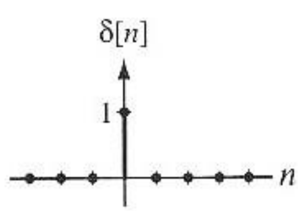
At n=0, x(0)= $(1/2)^0$ u(0-1)= 1.0 =**0**
At n=1, x(1)= $(1/2)^1$ u(1-1)= (½).1 =½
At n=2, x(2)= $(1/2)^2$ u(2-1)= (¼).1 =¼
At n=3, x(3)= $(1/2)^3$ u(3-1)= (1/8).1 =**1/8** & so on.



9) **Define discrete time unit step & unit impulse functions.** **[Nov '14]**

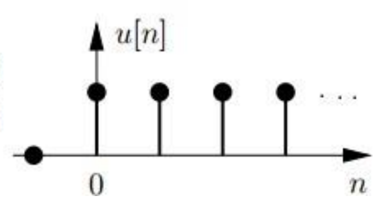
Unit Impulse(DT)

$$\delta[n] = \begin{cases} 1\ , n = 0 \\ 0\ , n \neq 0 \end{cases}$$



Unit Step(DT)

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



10) Define Energy & Power signals. **[Nov '14]**

For a signal x(t) or x(n), if energy is finite i.e., 0<E<∞ & power is zero i.e., P=0; then, that signal is called an Energy Signal.

For a signal x(t) or x(n), if power is finite i.e., 0<P<∞ & energy is infinite i.e., E=∞; then, that signal is called an Power Signal.

11) Find the value of the integral $\int e^{-2t} f(t+2)dt.$ **[Nov '15](R13)**

W.K.T, $\int x(t)\delta(t-t_o)dt = x(t_o)$ $\therefore \int e^{-2t} f(t+2)dt = e^{-2t}$ at t=-2

$$\therefore \int e^{-2t} f(t+2)dt = e^{-4}$$

12) Give the relation between continuous time unit impulse function f(t), step function u(t) & ramp function r(t). **[Nov '15](R13)**

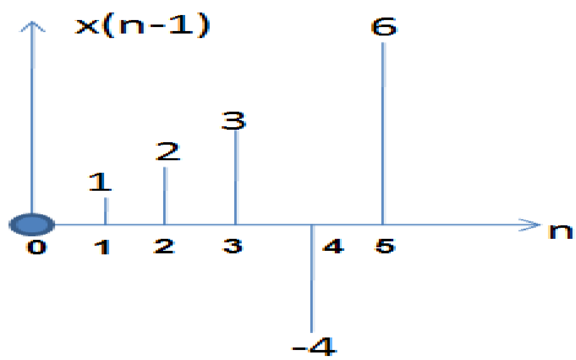
Continuous-time unit impulse and unit step functions:

$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}, \quad \text{and} \quad \delta(t) = \lim_{\Delta \downarrow 0} \delta_\Delta(t) \quad \text{where } \delta_\Delta(t) = \frac{u(t)-u(t-\Delta)}{\Delta}.$$

& **r(t) = t.u(t).** Also, $r(t) = \int u(t)dt = t$ & $u(t) = \frac{d}{dt} r(t)$

13) Given x(n) = {1,2,3,-4,6}; Plot the signal x(n-1). **[Nov '15]**

x(n-1) is **x(n)** delayed by one sample. i.e., **x(n-1)** = **{0,1,2,3,-4,6}**



14) Define Power signal. **[Nov '15]**

For a signal x(t) or x(n), if power is finite i.e., 0<P<∞ & energy is infinite i.e., E=∞; then, that signal is called an Power Signal.

15) Represent x(n)={1,-4,3,1,5,2} in terms of weighted shifted impulse functions. **[May ’14]**

$x(n) = \sum_{k=0}^{5} x(k)\delta(n-k)$ = 1.δ(n) -4.δ(n-1) +3.δ(n-2) +1.δ(n-3) +5.δ(n-4) +2.δ(n-5)

16) Define a random signal **[May ’15](R13)**

signal not defined by mathematical equation and arise random in nature . Eg noise

17) Define signal. **[May ’15]**

A **signal** is a physical quantity that varies with time, space or any other independent variable or variables.

18) What is meant by stability of a system? **[May ’15]**

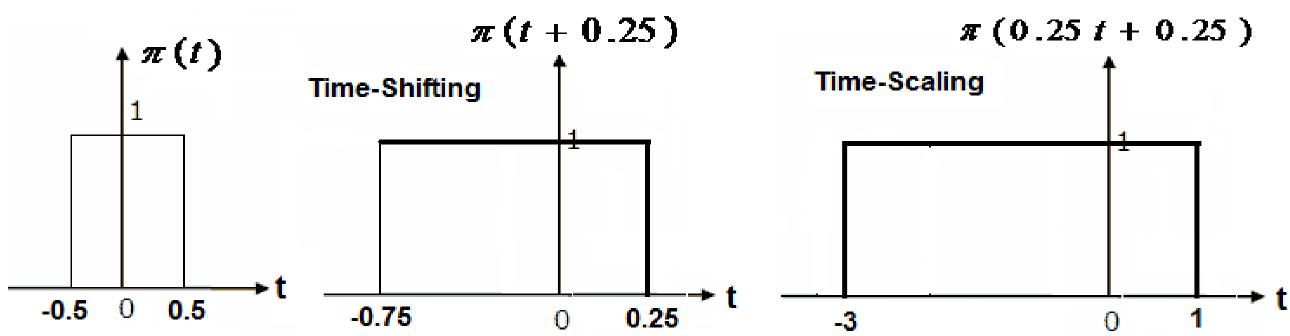
A system is said to be stable if a bounded input sequence always produces a bounded output sequence.

For an **LTI-CT system** to be **Stable**, $\int_{-\infty}^{\infty} |h(t)| dt < \infty$.

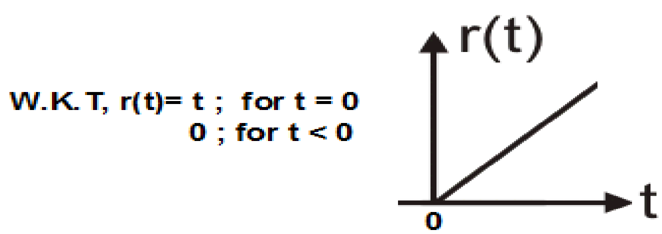
19) Sketch the following signals : $rect(\frac{t+1}{4})$; 5 $ramp(0.1t)$ **[May ’16](R13)**

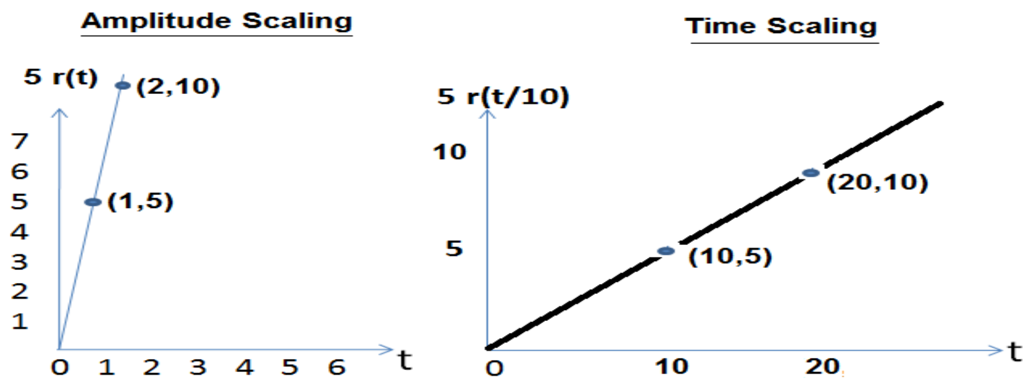
$rect(\frac{t+1}{4})$; 5 $ramp(0.1t)$

(i) $rect(\frac{t+1}{4}) = \pi(0.25t + 0.25)$



(ii) 5$ramp(0.1t)$ = 5 r(t/10)





20) **Given** $g(n) = 2e^{-2n-3}$. Write out & simplify the functions. **[May '16](R13)**

(i) $g(2-n)$ (ii) $g(\frac{n}{10}+4)$

(i) $g(2-n) = g(-n+2)$

Time-Shifting : $g(n+2) = 2e^{-2(n+2)-3} \quad = 2e^{-2n}.e^{-4}.e^{-3} = 2e^{-2n}.e^{-7}$

Time-Reversal : $g(2-n) = g(-n+2) \quad = 2e^{2n}.e^{-4}.e^{-3} = 2e^{2n}.e^{-7}$

(ii) $g(\frac{n}{10}+4) = g(0.1n+4)$

Time-Shifting : $g(n+4) = 2e^{-2(n+4)-3} = 2e^{-2n}.e^{-8}.e^{-3} = 2e^{-2n}.e^{-11}$

Time-Reversal : $g(0.1n+4) = 2e^{-2n/10}.e^{-8}.e^{-3} = 2e^{-0.2n}.e^{-11}$

21) **Check whether the DT signal sin3n is periodic.** **[May '16]**

N=2pi*n/w = 2*pi*n/3 not a ratio of interger hence not periodic

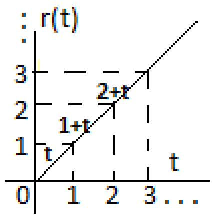
[May '16]

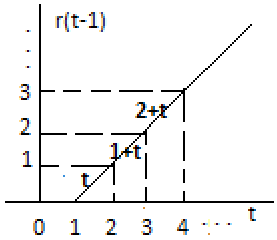
UNIT 1 CLASSIFICATION OF SIGNALS AND SYSTEMS

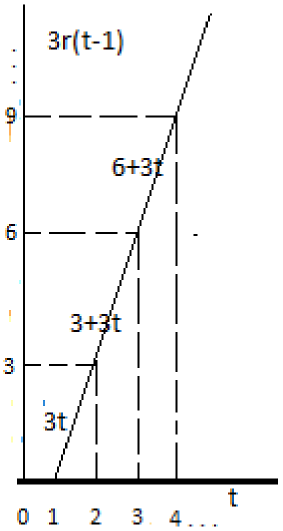
PART B

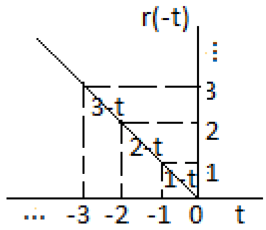
Q.1 Sketch $x(t) = 3r(t-1) + r(-t+2)$

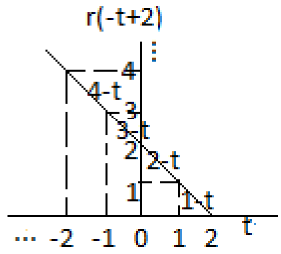
Draw time reversal signal of unit step signal Solution: $u(n) = 1;\ n \geq 0$





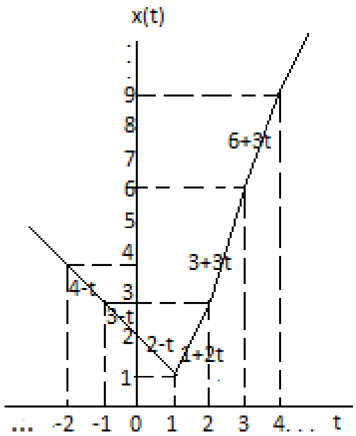






$$
\begin{aligned}
x(t) &= 3r(t-1) + r(-t+2) \\
&= 0 + 4 - t \ \ for -2 \leq t \leq -1 \\
&= 0 + 3 - t \ for -1 \leq t \leq 0 \\
&= 0 + 2 - t \ for\ 0 \leq t \leq 1 \\
&= 3t + 1 - t \ for\ 1 \leq t \leq 2 \\
&= 3 + 3t + 0 \ for\ 2 \leq t \leq 3 \\
&= 6 + 3t + 0 \ for\ 3 \leq t \leq 4
\end{aligned}
$$

and so on



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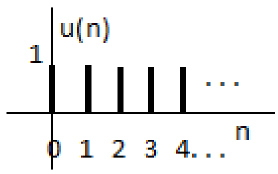
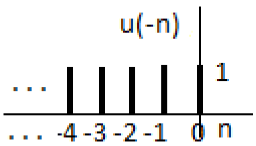




Q.2.Check whether the following is periodic or not. If periodic, determine fundamental time period

a. $\boldsymbol{x(t) = 2\cos(5t+1) - \sin(4t)}$

Here $\Omega_1 = 5,\ \Omega_2 = 4$

$$T_1 = \frac{2\pi}{\Omega_1} = \frac{2\pi}{5} = \frac{2\pi}{5}$$

$$T_2 = \frac{2\pi}{\Omega_2} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\frac{T_1}{T_2} = \frac{\frac{2\pi}{5}}{\frac{\pi}{2}} = \frac{4}{5} \text{(It is rational number)}$$

Hence $x(t)$ is **periodic**

$$T = 5T_1 = 4T_2 = 2\pi$$

$\therefore\ x(t)$ is **periodic** with period $\boldsymbol{2\pi}$

b. $x(n) = 3\cos 4\pi n + 2\sin \pi n$

Here $\omega_1 = 4\pi,\ \omega_2 = \pi$

$$N_1 = \frac{2\pi m}{\omega_1} = \frac{2\pi m}{4\pi} = \frac{m}{2}$$

$N_1 = 1$ (taking $m = 2$)

$$N_2 = \frac{2\pi m}{\omega_2} = \frac{2\pi m}{\pi} = 2m$$

$N_2 = 2$ (taking $m = 1$)

$$N = LCM(1,2) = 2$$

Hence x(n) $\therefore x(n)$ is **periodic** with period **2**

Q3. Determine whether the signals are energy or power signal

$$\boldsymbol{x(t) = e^{-3t}u(t)}$$

$$\boldsymbol{Energy\ E_\infty} = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2 dt = \lim_{T\to\infty}\int_{0}^{T}|e^{-3t}|^2 dt = \lim_{T\to\infty}\int_{0}^{T} e^{-6t}\, dt = \lim_{T\to\infty}\left[\frac{e^{-6t}}{-6}\right]_0^T$$

$$= \lim_{T\to\infty}\left[\frac{e^{-6T}}{-6} - \left[\frac{e^{-0}}{-6}\right]\right] = \frac{\boldsymbol{1}}{\boldsymbol{6}} < \infty \qquad\qquad \because e^{-\infty} = 0, e^{-0} = 1$$

$$\textbf{Power } \boldsymbol{P_\infty} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} |e^{-3t}|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_0^T e^{-6t}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-6t}}{-6}\right]_0^T = \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{-6T}}{-6} - \frac{e^{-0}}{-6}\right] = \lim_{T\to\infty} \frac{1}{2T} \left[\frac{1}{6}\right] = \mathbf{0} \qquad \because e^{-\infty} = 0, e^{-0} = 1, \frac{1}{\infty} = 0$$

Since **energy** value is **finite** and average **power** is **zero,** the given signal is an **energy** signal.

Q.4. Determine whether the signals are energy or power signal

$$\boldsymbol{x(n) = e^{j\left(\frac{\pi n}{4} + \frac{\pi}{2}\right)}}$$

$$\textbf{Energy } \boldsymbol{E_\infty} = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \sum_{n=-N}^{N} \left|e^{j\left(\frac{\pi n}{4} + \frac{\pi}{2}\right)}\right|^2 = \lim_{N\to\infty} \sum_{n=-N}^{N} 1^2 = \lim_{N\to\infty} 2N + 1 = \infty$$

$$\because \left|e^{j(\omega n + \theta)}\right| = 1 \ \ and \ \sum_{n=-N}^{N} 1 = 2N + 1$$

$$\textbf{Average power } \boldsymbol{P_\infty} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|e^{j\left(\frac{\pi n}{4} + \frac{\pi}{2}\right)}\right|^2$$

$$\lim_{=N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1^2 = \lim_{N\to\infty} \frac{1}{2N+1} (2N+1) = \mathbf{1}$$

Since **energy** value is **infinite** and average **power** is **finite,** the given signal is **power** signal

Q.5.Determine whether the following systems are linear or not

$$\boldsymbol{\frac{dy(t)}{dt} + ty(t) = x^2(t)}$$

Output due to weighted sum of inputs:

$$\frac{d[ay_1(t) + by_2(t)]}{dt} + t[ay_1(t) + by_2(t)] = [ax_1(t) + bx_2(t)]^2 \ldots (1)$$

Weighted sum of outputs:

For input $x_1(t)$:

$$\frac{dy_1(t)}{dt} + ty_1(t) = {x_1}^2(t) \ldots (2)$$

For input $x_2(t)$:

$$\frac{dy_2(t)}{dt} + ty_2(t) = {x_2}^2(t) \ldots (3)$$

$$(2) \times a + (3) \times b \Rightarrow a\frac{dy_1(t)}{dt} + aty_1(t) + b\frac{dy_2(t)}{dt} + bty_2(t) = a{x_1}^2(t) + b{x_2}^2(t) \ldots (4)$$

$$(1) \neq (4)$$

The given system is **Non-Linear**

Q.6.Determine whether the following systems are linear or not $\boldsymbol{y\ n.=x\ n-2.+x(n^2)}$

Output due to weighted sum of inputs:

$$y_3(n) = ax_1(n-2) + bx_2(n-2) + ax_1(n^2) + bx_2(n^2)$$

Weighted sum of outputs:
For input $x_1(n)$:

$$y_1(n) = x_1(n-2) + x_1(n^2)$$

For input $x_2(n)$:

$$y_2(n) = x_2(n-2) + x_2(n^2)$$
$$ay_1(n) + by_2(n) = ax_1(n-2) + ax_1(n^2) + bx_2(n-2) + bx_2(n^2)$$
$$\because y_3(n) = ay_1(n) + by_2(n)$$

Determine whether the following systems are static or dynamic $\boldsymbol{y\ t.=x\ 2t.+2x\ t.}$

$$y(0) = x(0) + 2x(0) \Rightarrow \text{ present inputs}$$
$$y(-1) = x(-2) + 2x(-1) \Rightarrow \text{ past and present inputs}$$
$$y(1) = x(2) + 2x(1) \Rightarrow \text{ future and present inputs}$$

Since output depends on past and future inputs the given system is **dynamic system**

Determine whether the following systems are static or dynamic
$\boldsymbol{y(n) = sinx(n)}$

$$y(0) = sinx(0) \quad \Rightarrow \text{ present input}$$
$$y(-1) = sinx(-1) \quad \Rightarrow \text{ present input}$$
$$y(1) = sinx(1) \quad \Rightarrow \text{ present input}$$

Since output depends on present input the given system is **Static system**

Q.7.Determine whether the following systems are time invariant or not $\boldsymbol{y(t) = x(t)sinwt}$
Output due to input delayed by T seconds

$$y(t,T) = x(t-T)sinwt$$

Output delayed by T seconds

$$y(t-T) = x(t-T)sinw(t-T)$$
$$\because\ y(t,T) \neq y(t-T)$$

The given system is **time variant**

Determine whether the following systems are time invariant or not
$\boldsymbol{y(n) = x(-n+2)}$
Output due to input delayed by k seconds

$$y(n,k) = x(-n+2-k)$$

Output delayed by k seconds

$$y(n-k) = x(-(n-k)+2) = x(-n+k+2)$$
$$\because\ \boldsymbol{y(n,k) \neq y(n-k)}$$

The given system is **time variant**

Q8 Determine whether the following systems are causal or not

$$\boldsymbol{y(t) = \frac{dx(t)}{dt} + 2x(t)}$$

The given equation is differential equation and the output depends on past input. Hence the given system is **Causal**

Determine whether the following systems are causal or not

$\boldsymbol{y(n) = sinx(n)}$

$$y(0) = sinx(0) \quad \Rightarrow \text{ present input}$$
$$y(-1) = sinx(-1) \quad \Rightarrow \text{ present input}$$
$$y(1) = sinx(1) \quad \Rightarrow \text{ present input}$$

Since output depends on present input the given system is **Causal system**

Determine whether the following systems are stable or not

$\boldsymbol{h(t) = e^{-4t}u(t)}$

Condition for stability $\int_{-\infty}^{\infty} |h(\tau)|d\tau < \infty$

$$\int_{-\infty}^{\infty} |h(\tau)|d\tau = \int_{-\infty}^{\infty} |(e^{-4\tau})u(\tau)|\, d\tau = \int_{0}^{\infty} e^{-4\tau} d\tau = \left[\frac{e^{-4\tau}}{-4}\right]_0^{\infty} = \frac{1}{4}$$

$\because \int_{-\infty}^{\infty} |h(\tau)|d\tau < \infty$ the given system is **stable**

Determine whether the following systems are stable or not $\boldsymbol{y\,n. = 3x(n)}$

$$\text{Let } x(n) = \delta(n), y(n) = h(n)$$
$$\Rightarrow h(n) = 3\delta(n)$$

Condition for stability $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=0}^{\infty} |3\delta(k)| = \sum_{k=0}^{\infty} 3\delta(k) = 3$$

$$\because \delta(k) = 0 \; for \; k \neq 0 \quad and \quad \delta(k) = 1 \; for \; k = 0$$

$\because \sum_{k=-\infty}^{\infty} |h(k)| < \infty$ the given system is **stable**

12 Determine and sketch even and odd part of the signal x(t)

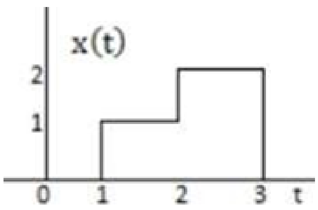
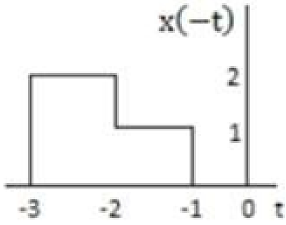


Fig: 1.82



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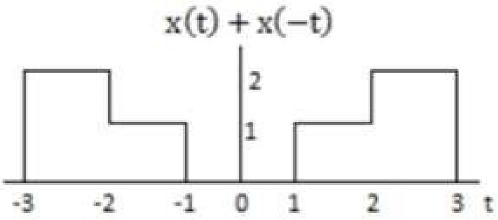

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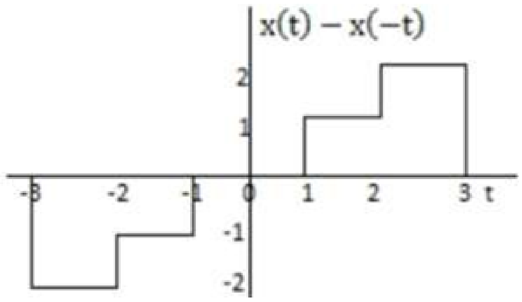

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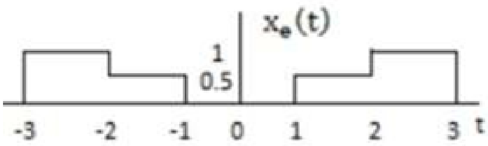


Fig: 1.86

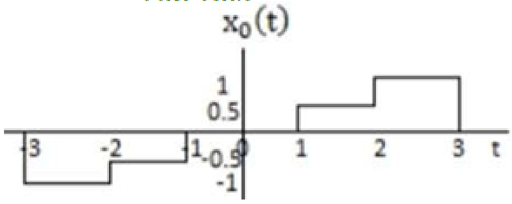


Fig: 1.87

UNIT II

PART A

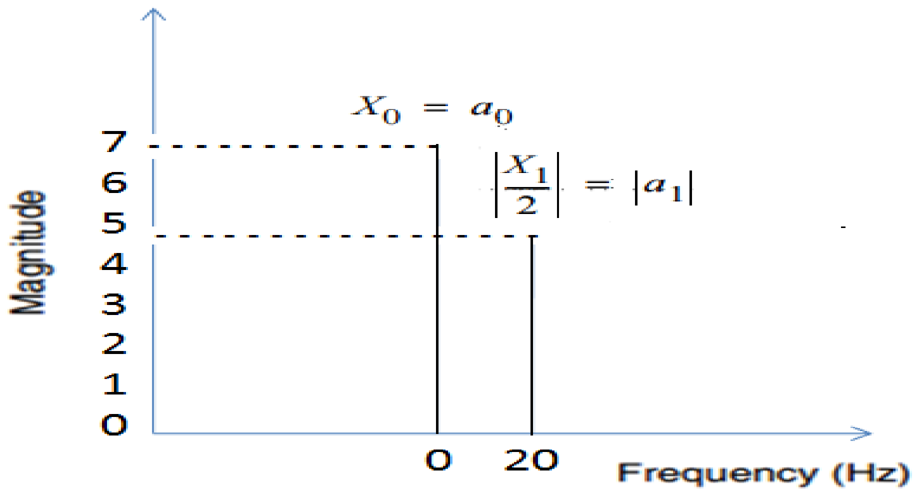
UNIT II : ANALYSIS OF CONTINUOUS TIME SIGNALS

1) Draw the single sided spectrum for x(t)=7 + 10cos[40πt+ π/2]. **[Nov '11]**

Expanding x(t) into complex sinusoid pairs,

$$x(t) = 7 + \frac{10}{2} e^{j(2\pi.20t + \pi/2)} + \frac{10}{2} e^{-j(2\pi.20t + \pi/2)}$$

The freq pairs that define the two-sided line spectrum are { (**0,7**), (**20,5e$^{j\pi/2}$**), (**20,5e$^{-j\pi/2}$**) }



2) What are the LTs of δ(t) & u(t)? **[Nov '11]**

LT[δ(t)] = **1** & LT[u(t)] = **1/s**

3) Give synthesis & analysis equations of CTFT. **[Nov '12]**

Analysis equation of CTFT

CTFT[x(t] = X(jΩ) = $\int x(t) e^{-j\Omega t} dt$

Synthesis equation of CTFT

Inv CTFT[X(jΩ)] = x(t) = $\frac{1}{2\pi}\int X(j\Omega) e^{j\Omega t} d\Omega$

4) Define the region of convergence of the LT. **[Nov '12]**

W.K.T, LT[x(t)] = X(s) exists if $\int_{-\infty}^{\infty} \left| x(t) e^{\sigma t} \right| < \infty$. The range of **σ** for which the **LT** converges is known as **ROC.**

5) State Dirichlet's conditions. **[Nov '13]**

The Fourier Transform does not exist for all periodic functions. The conditions for x(t) to have FT are; (i) x(t) is absolutely integrable over (-∞,∞) i.e.,

$\int |x(t)| dt < \infty$ (ii) x(t) has finite number of discontinuities & a finite number of maxima & minima in every finite time interval.

6) Give the equation for Trignometric fourier series. **[Nov '13]**

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\Omega_0 t) + b_n \sin(n\Omega_0 t)]$$

7) State the convergence of Fourier series rep'n of CT periodic sigs. **[Nov '14](R13)**

Most of the results presented for 2π-periodic functions extend easily to functions 2*L*-periodic functions. So we only discuss the case of 2π-periodic functions.

Definition. The function *f*(*x*) defined on [*a*,*b*], is said to be **piecewise continuous** if and only if, there exits a partition {x_1, x_2,… x_n} of [*a*,*b*] such that

(i) *f*(*x*) is continuous on [*a*,*b*] except may be for the points x_i,
(ii) The right-limit and left-limit of *f*(*x*) at the points x_i exist.

8) Find the ROC of the LT of x(t) = u(t). **[Nov '14](R13)**

$$\int x(t)e^{-st}dt = \int u(t)e^{-st}dt = \int_0^{\infty} e^{-st}dt = 1/s$$

<u>**ROC:**</u> $\sigma = \mathrm{Re}[s] > 0$

9) What is the relation between Fourier Transform & Laplace Transform? [Nov '14]

X(jΩ)=X(s) at s=jΩ $\int x(t)e^{-st}dt$ at s= jΩ $= \int x(t)e^{-j\Omega t}dt$

10) State Dirichlets conditions. **[Nov '14]**

Finite maxima,minima
absolutely integrable
minimum discontinuity **[Nov '15](R13)**

12) Give the relation between Fourier Transform & Laplace Transform.[Nov '15](R13)

X(k)= X(s)/when s=jw w

13) Obtain the fourier series coefficients for x(n)=sinω₀n.

$$x[n]=\sum_{k=(N)} a_k\, e^{jk\omega_0 n};\ \text{where, } \omega_0=\frac{2\pi}{N}$$

$$x[n]=\frac{1}{2j}e^{j(\frac{2\pi}{N})n}-\frac{1}{2j}e^{-j(\frac{2\pi}{N})n}$$

Comparing with the definition, we see that, the **fourier series coefficients** are

$$a_1=\frac{1}{2j},\ a_{-1}=-\frac{1}{2j}.$$

14) State the initial & final value theorems of LTs. **[May '11]**

- **Initial value theorem:** $$x(0)=\lim_{s\to\infty} sX(s)$$
- **Final value theorem:** $$x(\infty)=\lim_{s\to 0} sX(s)$$

15) Define Nyquist rate. **[May '12]**

The min rate at which a signal can be sampled & still be reconstructed from its samples is called the Nyquist rate. It is always equal to $2f_m$, where f_m is the max freq component present in the signal.

16) Determine the Fourier series coefficients for the signal cosπt. **[May '12]**

The complex Exponential Fourier series representation of a periodic signal with fundamental period T_0 is given by,

$$x(t)=\sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t}\ ;\ \text{where } \omega_0=\frac{2\pi}{T}$$

To evaluate the complex **Fourier coefficients** of **cosπt,** we can use Euler's formula.

i.e., $$\cos\pi t=\frac{1}{2}(e^{j\pi t}+e^{-j\pi t})=\frac{1}{2}e^{j\pi t}+\frac{1}{2}e^{-j\pi t}$$

Comparing with the **formula,** we get $c_1=\frac{1}{2}e^{j\pi}=\frac{-1}{2}$ &

$$c_{-1}=\frac{1}{2}e^{-j\pi}=\frac{-1}{2}$$

17) Determine the LT of the signal δ(t-5) & u(t-5). **[May '12]**

Using time-shifting property, $LT[x(t-\tau)]=e^{-s\tau}X(s)$

(i) $LT[\delta(t-5)]=e^{-5s}$ $\quad\because LT[\delta(t)]=X(s)=1$

(ii) $LT[u(t-5)]=\frac{e^{-5s}}{s}$ $\quad\because LT[u(t)]=X(s)=\frac{1}{s}$

18) State the Time-Scaling property of LT. **[May ’13]**

$$LT[x(at)] = \frac{1}{|a|} X\left(\frac{s}{a}\right)$$

19) What is the FT of a DC signal of amplitude 1? **[May ’13]**

FT[1] = 2πδ(Ω) $\because F^{-1}[\delta(\Omega)] = \frac{1}{2\pi}$

20) State the conditions for convergence of fourier series. **[May ’14]**

Refer **Q.No.7 ; Nov 2014(R13)**

21) State any 2 properties of ROC of LT X(s) of a signal x(t). **[May ’14]**

(i) $LT[x(t-\tau)] = e^{-s\tau} X(s)$ **(ii)** $LT[x(at)] = \frac{1}{|a|} X\left(\frac{s}{a}\right)$

22) Find the Fourier coefficients of x(t)=1+sin2ωt+2cos2ωt+cos[3ωt+π/3). **[May’15](R13)**

Expanding x(t) in terms of complex exponentials,

$$x(t) = 1 + \frac{1}{2j}(e^{j2\omega_0 t} - e^{-j2\omega_0 t}) + 2.\frac{1}{2}(e^{j2\omega_0 t} + e^{-j2\omega_0 t}) + \frac{1}{2}(e^{j[3\omega_0 t + \pi/3]} + e^{j[3\omega_0 t + \pi/3]})$$

Collecting terms we get,

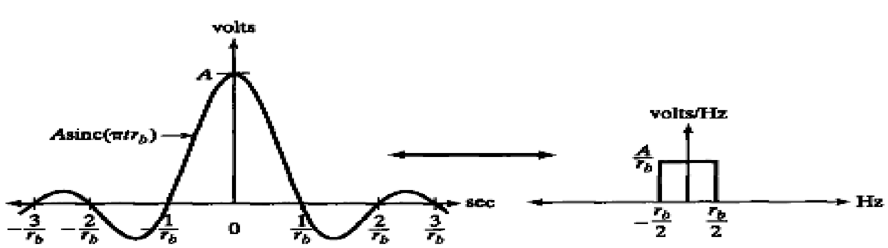
$$x(t) = 1 + \left(1 + \frac{1}{2j}\right)e^{j2\omega_0 t} + \left(1 - \frac{1}{2j}\right)e^{-j2\omega_0 t} + \left(\frac{1}{2}e^{j(\pi/3)}\right)e^{j3\omega_0 t} + \left(\frac{1}{2}e^{-j(\pi/3)}\right)e^{-j3\omega_0 t}$$

Thus, the **Fourier coefficients** are, $c_0=1$, $c_1 = 1 + \frac{1}{2j} = 1 - \frac{1}{2}j$, $c_{-1} = 1 - \frac{1}{2j} = 1 + \frac{1}{2}j$,

$c_2 = \frac{1}{2}e^{(j\pi/4)} = \frac{\sqrt{2}}{4}(1+j)$, $c_{-2} = \frac{1}{2}e^{-(j\pi/4)} = \frac{\sqrt{2}}{4}(1-j)$

23) Draw the spectrum of a CT rectangular pulse. **[May ’15](R13)**

The spectrum of a CT rectangular pulse is a sinc function.



24) Given x(t)=δ(t);find X(s) & X(ω). **[May '15] (R13)**

$X(s) = LT[x(t)] = 1$ & $X(\omega) = FT[x(t)] = 1$

25) State the relationship between FT & LT. **[May '15]**

Refer **Q.No.9 ; Nov 2014**

26) Find the fourier series coefficients of the signal x(t)=sinωt. **[May '15]**

Refer **Q.No.10 ; May 2015(R13),** we get $c_1 = \frac{1}{2j}$ & $c_{-1} = -\frac{1}{2j}$.

27) What is the inverse Fourier Transform of (i) $e^{-j2\pi f t_o}$ **(ii)** $\delta(f - f_o)$ **[May '16](R13)**

(i) $x(t) = \frac{1}{2\pi}\int X(j\Omega)e^{j\Omega t}d\Omega$

$x(t) = \frac{1}{2\pi}\int e^{-j2\pi\Omega t_0} e^{j\Omega t}d\Omega = \infty$

(ii) $x(t) = \frac{1}{2\pi}\int X(j\Omega)e^{j\Omega t}d\Omega$

$x(t) = \frac{1}{2\pi}\int \delta(\Omega - \Omega_0)e^{j\Omega t}d\Omega = \frac{1}{2\pi}e^{j\Omega_0 t}$ (or) $F^{-1}[2\pi\delta(\Omega - \Omega_0)] = \frac{1}{2\pi}e^{j\Omega_0 t}$

28) Give the Laplace Transform of $x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$ with ROC. **[May '16] (R13)**

$X(s) = \frac{3}{s+2} - \frac{2}{s+1}$; $\text{ROC} : \sigma > -1$ $\because L[e^{-at}u(t)] = \frac{1}{s+a}$

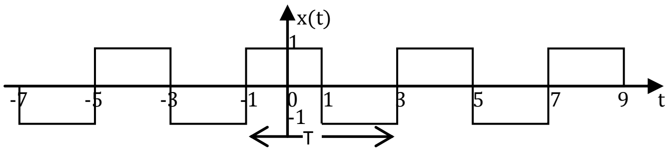
219) Determine the LT of the signal δ(t-5) & u(t-5).

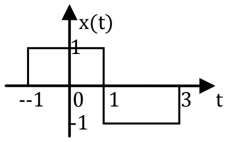
(e^-5s) & (e^-5s)/s

[May '16]

UNIT II CONTINUOUS TIME SIGNAL ANALYSIS
PART B

Q.1 Find the trigonometric Fourier series for the periodic signal x(t) as shown in Figure





Solution:

$T = 3-(-1) = 4 \; and \; \Omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

Evaluation of a_0

$$\boldsymbol{a_0} = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)dt = \frac{1}{4}\left[\int_{-1}^{1} 1\,dt + \int_{1}^{3} -1\,dt\right] = \frac{1}{4}\left[[t]_{-1}^{1} - 1[t]_1^3\right] = \frac{1}{4}[(1-(-1))-(3-1)]$$

$$= \frac{1}{4}[2-2] = \mathbf{0}$$

Evaluation of a_n

$$\boldsymbol{a_n} = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\cos n\Omega_0 t\,dt = \frac{2}{4}\left[\int_{-1}^{1} \cos n\Omega_0 t\,dt + \int_{1}^{3}(-1)\cos n\Omega_0 t\,dt\right]$$

$$= \frac{1}{2}\left[\left[\frac{\sin n\Omega_0 t}{n\Omega_0}\right]_{-1}^{1} - \left[\frac{\sin n\Omega_0 t}{n\Omega_0}\right]_{1}^{3}\right] = \frac{1}{2}\left[\left[\frac{\sin n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{-1}^{1} - \left[\frac{\sin n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{1}^{3}\right]$$

$$= \frac{1}{2}\left(\frac{2}{n\pi}\right)\left[\sin n\frac{\pi}{2} - \left(\sin n\frac{\pi}{2}(-1)\right) - \left(\sin n\frac{\pi}{2}(3) - \sin n\frac{\pi}{2}\right)\right]$$

$$= \left[\frac{1}{n\pi}\right]\left[\sin n\frac{\pi}{2} + \sin n\frac{\pi}{2} - \sin 3n\frac{\pi}{2} + \sin n\frac{\pi}{2}\right] = \frac{1}{n\pi}\left[3\sin\frac{n\pi}{2} - \sin(2n\pi - \frac{n\pi}{2})\right]$$

$$= \frac{1}{n\pi}\left[3\sin\frac{n\pi}{2} - \left(-\sin n\frac{\pi}{2}\right)\right] = \boldsymbol{\frac{4}{n\pi}\left[\sin n\frac{\pi}{2}\right]}$$

Evaluation of b_n

$$\boldsymbol{b_n} = \frac{2}{T}\int_{t_0}^{t_0+T} x(t)\sin n\Omega_0 t\,dt = \frac{2}{4}\left[\int_{-1}^{1} \sin n\Omega_0 t\,dt + \int_{1}^{3} -\sin n\Omega_0 t\,dt\right]$$

$$= \frac{1}{2}\left[\left[\frac{-\cos n\Omega_0 t}{n\Omega_0}\right]_{-1}^{1} - \left[\frac{-\cos n\Omega_0 t}{n\Omega_0}\right]_{1}^{3}\right] = \frac{1}{2}\left[\left[\frac{-\cos n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{-1}^{1} + \left[\frac{\cos n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{1}^{3}\right]$$

$$= \frac{1}{2}\left[\frac{-2}{n\pi}\left(\cos n\frac{\pi}{2} - \cos n\frac{\pi}{2}(-1)\right) + \frac{2}{n\pi}\left(\cos n\frac{\pi}{2}(3) - \cos n\frac{\pi}{2}\right)\right]$$

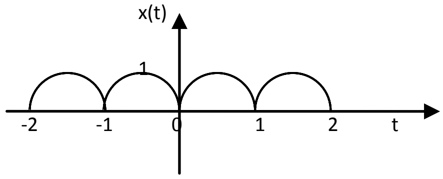
$$= \frac{1}{2}\left[0 + \frac{2}{n\pi}\left(\cos\left(2n\pi - \frac{n\pi}{2}\right) - \cos n\frac{\pi}{2}\right)\right] = \left[\frac{1}{n\pi}\left(\cos n\frac{\pi}{2} - \cos n\frac{\pi}{2}\right)\right] = \mathbf{0}$$

Trigonometric Fourier series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\Omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\Omega_0 t$$

$$= \sum_{n=1}^{\infty} \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos n\Omega_0 t = \sum_{n=1}^{\infty} \frac{4}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos n\frac{\pi}{2}t$$

Q.2 **Obtain Fourier series of the following full wave rectified sine wave shown in figure**



Solution:

$$x(t) = x(-t); \quad \therefore \textit{Given signal is even signal, so } \boldsymbol{b_n = 0}$$

$$T = 1 \text{ and } \Omega_0 = \frac{2\pi}{1} = 2\pi$$

The given signal is sinusoidal signal, $\therefore x(t) = A \sin \Omega t$

$$\text{Here} \quad \Omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \quad \text{and } A = 1$$

$$\therefore \boldsymbol{x(t) = \sin \pi t}$$

Evaluation of a_0

$$\boldsymbol{a_0} = \frac{2}{T}\int_0^{\frac{T}{2}} x(t)\,dt = \frac{2}{1}\int_0^{\frac{1}{2}} x(t)\,dt = \left[2\int_0^{\frac{1}{2}} \sin \pi t\ \,dt\right] = 2\left[-\frac{\cos \pi t}{\pi}\right]_0^{\frac{1}{2}} = -\frac{2}{\pi}\left[\cos\frac{\pi}{2} - \cos 0\right] = \boldsymbol{\frac{2}{\pi}}$$

Evaluation of a_n

$$\boldsymbol{a_n} = \frac{4}{T}\int_0^{\frac{T}{2}} x(t)\cos n\Omega_0 t\,dt = \frac{4}{1}\int_0^{\frac{1}{2}} \sin \pi t \cos n2\pi t\ \,dt = 2\int_0^{\frac{1}{2}} [\sin((1+2n)\pi t) + \sin((1-2n)\pi t)]\,dt$$

$$= 2\left[-\frac{\cos((1+2n)\pi t)}{(1+2n)\pi} - \frac{\cos((1-2n)\pi t)}{(1-2n)\pi}\right]_0^{\frac{1}{2}}$$

$$= \frac{2}{\pi}\left[-\frac{\cos\left((1+2n)\frac{\pi}{2}\right)}{1+2n} - \frac{\cos\left((1-2n)\frac{\pi}{2}\right)}{1-2n} + \frac{1}{1+2n} + \frac{1}{1-2n}\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{1+2n} + \frac{1}{1-2n}\right] = \frac{2}{\pi}\left[\frac{1-2n+1+2n}{1-4n^2}\right] = \boldsymbol{\frac{4}{\pi(1-4n^2)}}$$

Trigonometric Fourier series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\Omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\Omega_0 t$$

$$\therefore x(t) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4}{\pi(1-4n^2)} \cos n2\pi t$$

2.3 Exponential Fourier series

The exponential form of Fourier series of a periodic signal $x(t)$ with period T is defined as,

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\,\Omega_0 t}$$

The Fourier coefficient c_n can be evaluated using the following formulae

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-jn\,\Omega_0 t}\,dt$$

Q.3 Find exponential series for the signal shown in figure

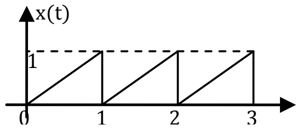


Fig 2.26

Solution:

$T = 1, \Omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$

Consider the equation of a straight line

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} \quad \dots\dots\dots \langle 9 \rangle$$

Consider one period of the given signal Fig 2.26 as shown in Fig 2.27

Consider points P,Q as shown in fig 2.27

Coordinates of point $P = [0,0]$

Coordinates of point $Q = [1,1]$

On substituting the coordinates of points P and Q in eq $\langle 9 \rangle$

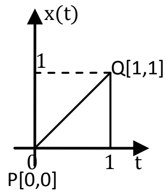


Fig 2.27

$$\frac{x(t) - 0}{1 - 0} = \frac{t - 0}{1 - 0} \Rightarrow x(t) = t$$

$$[\because x = t, y = x(t)]$$

Evaluation of c_0

$$c_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{1}\int_0^1 (t)dt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$$

Evaluation of c_n

$$c_n = \frac{1}{T}\int_0^T x(t)e^{-jn\,\Omega_0 t}dt = \frac{1}{1}\int_0^1 te^{-jn\,2\pi t}\,dt = \left[t\frac{e^{-jn\,2\pi t}}{-jn2\pi}\right]_0^1 - \int_0^1 \frac{e^{-jn\,2\pi t}}{-jn2\pi}dt$$

$$= \frac{e^{-jn\,2\pi}}{-jn2\pi} + 0 + \left[\frac{e^{-jn\,2\pi t}}{-j^2(n2\pi)^2}\right]_0^1 = j\frac{e^{-jn\,2\pi}}{n2\pi} + \frac{e^{-jn\,2\pi}}{n^2 4\pi^2} - \frac{1}{n^2 4\pi^2}$$

$$= \frac{j}{n2\pi} + \frac{1}{n^2 4\pi^2} - \frac{1}{n^2 4\pi^2} = \frac{j}{n2\pi}$$

$$c_n = \frac{j}{n2\pi}$$

$c_1 = \frac{j}{2\pi}$, $c_2 = \frac{j}{4\pi}$, $c_3 = \frac{j}{6\pi}$, $c_{-1} = \frac{j}{-2\pi}$, $c_{-2} = \frac{j}{-4\pi}$, $c_{-3} = \frac{j}{-6\pi}$

Exponential Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\,\Omega_0 t}$$

$$\therefore\ x(t) = +\cdots - \frac{j}{6\pi}e^{-j6\pi t} - \frac{j}{4\pi}e^{-j4\pi t} - \frac{j}{2\pi}e^{-j2\pi t} + \frac{1}{2} + \frac{j}{2\pi}e^{j2\pi t} + \frac{j}{4\pi}e^{j4\pi t} + \frac{j}{6\pi}e^{j6\pi t} + \cdots$$

$$= \frac{1}{2} + \frac{j}{2\pi}\left[e^{j2\pi t} - e^{-j2\pi t}\right] + \frac{j}{4\pi}\left[e^{j4\pi t} - e^{-j4\pi t}\right] + \frac{j}{6\pi}\left[e^{j6\pi t} - e^{-j6\pi t}\right] + \cdots$$

$$= \frac{1}{2} + \frac{1}{\pi}\left[\frac{e^{j2\pi t} - e^{-j2\pi t}}{(-1)2j}\right] + \frac{1}{2\pi}\left[\frac{e^{j4\pi t} - e^{-j4\pi t}}{(-1)2j}\right] + \frac{1}{3\pi}\left[\frac{e^{j6\pi t} - e^{-j6\pi t}}{(-1)2j}\right]$$

$$= \frac{1}{2} + \left(\frac{-1}{\pi}\right) sin\, 2\pi t - \frac{1}{2\pi} sin\, 4\pi t - \frac{1}{3\pi}\sin 6\pi t$$

$$= \frac{1}{2} - \frac{1}{\pi}\left[sin\, 2\pi t + \frac{1}{2} sin\, 4\pi t + \frac{1}{3} sin\, 6\pi t + \cdots\right]$$

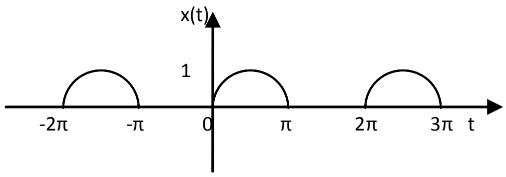
2.4 Cosine Fourier series

Cosine representation of $x(t)$ is

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n cos\,(n\Omega_0 t + \theta_n)$$

Where A_0 is dc component, A_n is harmonic amplitude or spectral amplitude and θ_n is phase coefficient or phase angle *or spectral angle*

Q.4 Determine the cosine Fourier series of the signal shown in Figure



Solution:

The signal shown in is periodic with period $T = 2\pi$ *and* $\Omega_0 = \frac{2\pi}{2\pi} = 1$

The given signal is sinusoidal signal, $\therefore x(t) = A \sin \Omega t$

$$\text{Here} \quad \Omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1, \ \ A = 1$$

$$\boldsymbol{\therefore x(t) = sin\, t}$$

Evaluation of a_0

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2\pi}\int_0^{\pi} sin\, t\, dt = \frac{1}{2\pi}[-cos\, t\,]_0^{\pi} = \frac{1}{2\pi}[-cos\,\pi + cos\, 0] = \frac{1}{2\pi}[2] = \frac{1}{\pi}$$

Evaluation of a_n

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\Omega_0 t\,dt = \frac{2}{2\pi}\int_0^\pi \sin t\ \cos nt\,dt = \frac{1}{2\pi}\int_0^\pi [\sin(1+n)t + \sin(1-n)t]\,dt$$

$$= \frac{1}{2\pi}\left[-\frac{\cos(1+n)t}{(1+n)} - \frac{\cos(1-n)t}{(1-n)}\right]_0^\pi$$

$$= \frac{1}{2\pi}\left[-\frac{\cos(1+n)\pi}{(1+n)} - \frac{\cos(1-n)\pi}{(1-n)} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

$$for\ n = odd:\ a_n = \frac{1}{2\pi}\left[-\frac{1}{1+n} - \frac{1}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right] = 0$$

$$for\ n = even:\ a_n = \frac{1}{2\pi}\left[\frac{1}{1+n} + \frac{1}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right] = \frac{1}{2\pi}\left(\frac{2}{1+n} + \frac{2}{1-n}\right)$$

$$= \frac{1}{\pi}\left[\frac{1-n+1+n}{1-n^2}\right] = \frac{2}{\pi(1-n^2)}$$

$$\therefore \boldsymbol{a_n} = \begin{cases} \mathbf{0} & \boldsymbol{for\ n = odd} \\ \dfrac{\mathbf{2}}{\boldsymbol{\pi(1-n^2)}} & \boldsymbol{for\ n = even} \end{cases}$$

Evaluation of b_n

$$\boldsymbol{b_n} = \frac{2}{T}\int_0^T x(t)\sin n\Omega_0 t\ dt = \frac{2}{2\pi}\int_0^\pi \sin t\ \sin nt\ dt = \frac{1}{2\pi}\int_0^\pi (\cos(1-n)t - \cos(1+n)t)\,dt$$

$$= \frac{1}{2\pi}\left[\frac{\sin(1-n)t}{(1-n)} - \frac{\sin(1+n)t}{(1+n)}\right]_0^\pi = \frac{1}{2\pi}\left[\frac{\sin(1-n)\pi}{(1-n)} - \frac{\sin(1+n)\pi}{(1+n)} - 0\right] = \mathbf{0}$$

Evaluation of Fourier coefficients of Cosine Fourier series from Trigonometric Fourier series:

$$A_0 = a_0 = \frac{1}{\pi}$$

$$A_n = \sqrt{{a_n}^2 + {b_n}^2} = \frac{2}{\pi(1-n^2)},\ for\ n = even$$

$$\theta_n = -\tan^{-1}\frac{b_n}{a_n} = 0$$

Cosine Fourier series

$$\mathrm{x(t) = A_0 + \sum_{n=1}^{\infty} A_n\cos(n\Omega_0 t + \theta_n)}$$

$$\therefore x(t) = \frac{1}{\pi} + \sum_{\substack{n=1 \\ (n=even)}}^{\infty} \frac{2}{\pi(1-n^2)}\cos nt = \frac{1}{\pi} + \frac{2}{\pi(1-4)}\cos 2t + \frac{2}{\pi(1-16)}\cos 4t + \cdots$$

$$= \frac{1}{\pi} - \frac{2}{3\pi}\cos 2t - \frac{2}{15\pi}\cos 4t + \cdots = \frac{1}{\pi} - \frac{2}{\pi}[\frac{1}{3}\cos 2t + \frac{1}{15}\cos 4t + \cdots\ ]$$

Fourier transform

The Fourier representation of periodic signals has been extended to non-periodic signals by letting the fundamental period T tend to infinity and this Fourier method of representing non-periodic signals as a function of frequency is called Fourier transform.

The Fourier transform (FT) of Continuous time signals is called Continuous Time Fourier Transform

$$Let\ x(t) = Continuous\ time\ signal$$
$$X(j\Omega) = F\{x(t)\}$$

The Fourier transform of continuous time signal, x(t) is defined as,

$$X(j\Omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t}\, dt$$

Conditions for existence of Fourier transform

The Fourier transform $x(t)$ exist if it satisfies the following Dirichlet condition

1. $x(t)$ should be absolutely integrable

$$ie\ , \qquad \int_{-\infty}^{\infty} x(t)dt < \infty$$

2. $x(t)$ should have a finite number of maxima and minima with in any finite interval.
3. $x(t)$ should have a finite number of discontinuities with in any interval.

Definition of Inverse Fourier Transform

The inverse Fourier Transform of $X(j\Omega)$ is defined as,

$$x(t) = F^{-1}\{X(j\Omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t}\, d\Omega$$

Q.5 Find Fourier transform of impulse signal

Solution:

By definition of Fourier transform

$$F\{x(t)\} = X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t}\, dt$$

$$\therefore F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j\,\Omega t}\, dt$$

$$F[\delta(t)] = \delta(0) e^{-j\,\Omega(0)} = 1 \qquad \left[\because\ Impulse\ signal\ \delta(t) = \begin{cases} 1\ for\ t = 0 \\ 0\ for\ t \neq 0 \end{cases}\right]$$

Find Fourier transform of double sided exponential signal

Solution:

Double sided exponential signal is given by

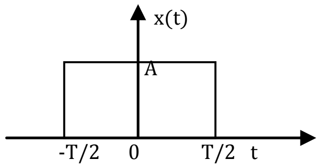
$$F[e^{-a|t|}] = \begin{cases} e^{-at} & : t \geq 0 \\ e^{at} & : t \leq 0 \end{cases}$$

$$F[e^{-a|t|}] = \int_{-\infty}^{0} e^{at} e^{-j\,\Omega t}\, dt + \int_{0}^{\infty} e^{-at} . e^{-j\,\Omega t}\, dt = \int_{-\infty}^{0} e^{(a-j\,\Omega)t} dt + \int_{0}^{\infty} e^{-(a+j\,\Omega)t} dt$$

$$= \left[\frac{e^{(a-j\,\Omega)t}}{a - j\,\Omega}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\,\Omega)t}}{-(a + j\,\Omega)}\right]_{0}^{\infty} = \frac{1}{a - j\,\Omega} + \frac{1}{a + j\,\Omega} = \frac{a + j\,\Omega + a - j\,\Omega}{a^2 + \Omega^2}$$

$$= \frac{2a}{a^2 + \Omega^2}$$

Q.6 **Find Fourier transform of rectangular pulse function shown in figure**



Solution:

$$x(t) = \pi(t) = A \quad ; \frac{-T}{2} \le t \le \frac{T}{2}$$

$$F[\pi(t)] = \int_{-\frac{T}{2}}^{\frac{T}{2}} Ae^{-j\Omega t}\,dt = A\left[\frac{e^{-j\Omega t}}{-j\Omega}\right]_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{A}{-j\Omega}[e^{-j\Omega\frac{T}{2}} - e^{j\Omega\frac{T}{2}}] = \frac{2A}{j\Omega}[\frac{e^{j\Omega\frac{T}{2}} - e^{-j\Omega\frac{T}{2}}}{2}] = \frac{2A}{\Omega}\sin\Omega\frac{T}{2}$$

$$= \frac{2A}{\Omega T}T\sin\Omega\frac{T}{2} = AT\frac{\sin\Omega\frac{T}{2}}{\Omega\frac{T}{2}} AT\sin c\,\Omega\frac{T}{2}$$

Find inverse Fourier transform $X(j\,\Omega) = \delta(\Omega)$

Solution:

$$\therefore F^{-1}[X(j\,\Omega)] = F^{-1}[\delta(\Omega)]$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\,\Omega)e^{j\,\Omega t}\,d\Omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\,\Omega)e^{j\,\Omega t}\,d\Omega = \frac{1}{2\pi}[1] \;\because\; \delta(\Omega) = \begin{cases} 1 \; for\,\Omega = 0 \\ 0 \; for\,\Omega \ne 0 \end{cases}$$

$$F^{-1}[\delta(\Omega)] = \frac{1}{2\pi}$$

2.6 Laplace transform

It is used to transform a time domain to complex frequency domain signal(s-domain)

Two Sided Laplace transform (or) Bilateral Laplace transform

Let $x(t)$ be a continuous time signal defined for all values of t. Let $X(S)$ be Laplace transform of $x(t)$.

$$L\{x(t)\} = X(S) = \int_{-\infty}^{\infty} x(t)e^{-St}\,dt$$

One sided Laplace transform (or) Unilateral Laplace transform

Let $x(t)$ be a continuous time signal defined for $t \ge 0$ (ie If $x(t)$ is causal) then,

$$L\{x(t)\} = X(S) = \int_{0}^{\infty} x(t)e^{-St}\,dt$$

Inverse Laplace transform

The S-domain signal $X(S)$ can be transformed to time domain signal x(t) by using inverse Laplace transform.

$$L^{-1}\{X(s)\} = x(t) = \frac{1}{2\pi j}\int_{s=\sigma-j\Omega}^{s=\sigma+j\Omega} X(S)e^{st}\, ds$$

Existence of Laplace transform

The necessary and sufficient conditions for the existence of Laplace transform are

- $x(t)$ should be continuous in the given closed interval
- $x(t)e^{-\sigma t}$ must be absolutely intergrable
  $i.e., X(S)\ exists\ only\ if\ \int_{-\infty}^{\infty}|x(t)e^{-\sigma t}|dt < \infty$

Q.7 **Find unilateral Laplace transform for the following signals**

$$\boldsymbol{i)\ \ x(t) = \delta(t)}$$

$$\mathrm{X(S)} = \int_0^\infty \mathrm{x(t)e^{-st}dt} = \int_0^\infty \mathrm{\delta(t)e^{-st}dt} = \mathrm{e^{-s(0)}} = 1 \qquad \because \delta(\mathrm{t}) = \begin{cases} 1 & \text{for t} = 0 \\ 0 & \text{for t} \neq 0 \end{cases}$$

$$\boldsymbol{ii)\ \ x(t) = u(t)}$$

$$X(S) = \int_0^\infty \mathrm{x(t)e^{-st}dt} = \int_0^\infty \mathrm{u(t)e^{-st}dt} = \int_0^\infty 1e^{-st}dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{1}{s} \qquad \because \mathrm{u(t)} = \begin{cases} 1 & \text{for t} \geq 0 \\ 0 & \text{for t} < 0 \end{cases}$$

Find Laplace transform of xntd $= e^{at}u(t)$

Solution:

$$X(S) = L[e^{at}u(t)] = \int_0^\infty e^{at}e^{-st}\,dt = \int_0^\infty e^{-(s-a)t}\,dt = \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = \frac{1}{s-a}$$

Q.8 **Determine initial value and final value of the following signal $X(S) = \frac{1}{s(s+2)}$**

Solution:

Initial value

$$x(0) = \operatorname*{Lt}_{s\to\infty} SX(S) = \operatorname*{Lt}_{s\to\infty} s\frac{1}{s(s+2)} = \frac{1}{\infty} = 0$$

Final value

$$x(\infty) = \operatorname*{Lt}_{s\to 0} SX(S) = \operatorname*{Lt}_{s\to 0} s\frac{1}{s(s+2)} = \frac{1}{2}$$

Q.9 **Find inverse Laplace Transform of XnSd $=\frac{S^2+9S+1}{S[S^2+6S+8]}$.** Find ROC for $i)\ Re(s) > 0$ $ii)\ Re(s) < -4$ $iii)\ -2 > Re(s) > -4$

Solution:

$$X(S) = \frac{S^2 + 9S + 1}{S[S^2 + 6S + 8]} = \frac{S^2 + 9S + 1}{S(S+4)(S+2)} = \frac{A}{S} + \frac{B}{(S+4)} + \frac{C}{(S+2)}$$

$$S^2 + 9S + 1 = A(S+4)(S+2) + BS(S+2) + CS(S+4)$$

at $S = 0$	$S = -4$	$S = -2$
$A = \frac{1}{8}$	$B = -\frac{19}{8}$	$C = \frac{13}{4}$

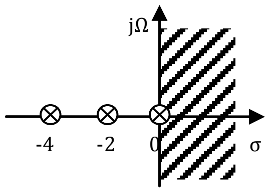
$$\therefore X(S) = \frac{\frac{1}{8}}{S} + \frac{-\frac{19}{8}}{(S+4)} + \frac{\frac{13}{4}}{(S+2)}$$

Applying inverse Laplace transform

$$x(t) = \frac{1}{8}u(t) - \frac{19}{8}e^{-4t}u(t) + \frac{13}{4}e^{-2t}u(t)$$

ROC

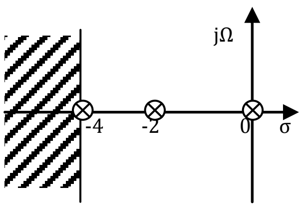
$i)\ \boldsymbol{Re(s) > 0}$



ROC lies right side of all poles

$$\therefore x(t) = \frac{1}{8}u(t) - \frac{19}{8}e^{-4t}u(t) + \frac{13}{4}e^{-2t}u(t)$$

$ii)\ \boldsymbol{Re(s) < -4}$

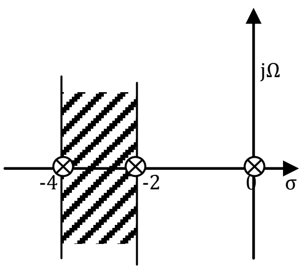


ROC lies left side of all poles

$$\therefore x(t) = -\frac{1}{8}u(-t) + \frac{19}{8}e^{-4t}u(-t) - \frac{13}{4}e^{-2t}u(-t)$$

$iii)\ -\boldsymbol{2} > Re(\boldsymbol{s}) > -\boldsymbol{4}$

ROC lies left side of poles s=-2, s=0 and right side of pole s=-4



$$\therefore x(t) = -\frac{1}{8}u(-t) - \frac{19}{8}e^{-4t}u(t) - \frac{13}{4}e^{-2t}u(-t)$$

UNIT III

PART A

UNIT III : LINEAR TIME INVARIANT- CONTINUOUS TIME SYSTEMS

1) Given the system impulse response h(t); state the conditions for causality & stability. **[Nov '11]**

For an **LTI-CT system** to be **causal**, **h(t)** = **0** for t < 0.

For an **LTI-CT system** to be **Stable**, $\int_{-\infty}^{\infty} h(t)dt < \infty$.

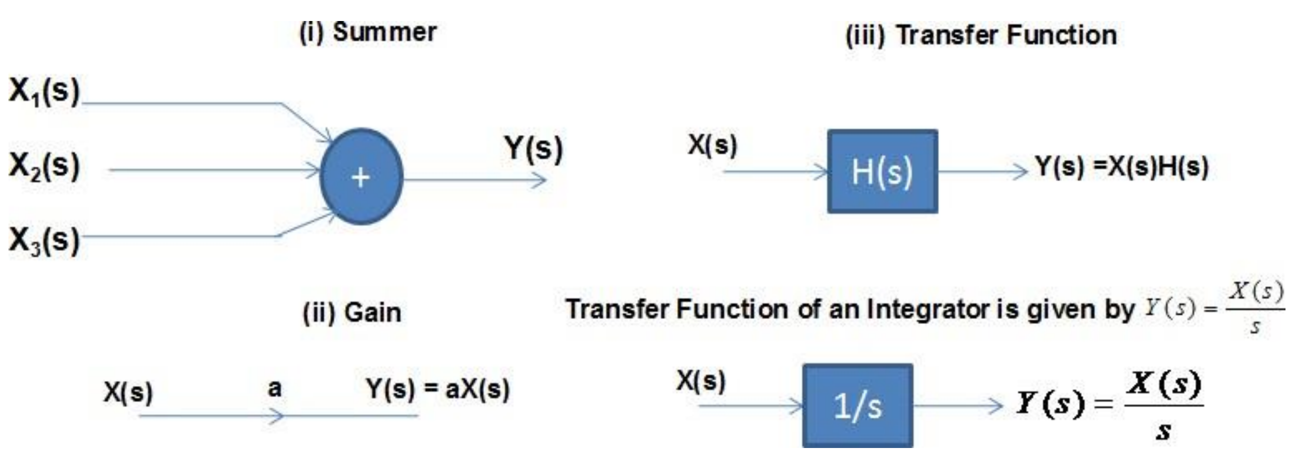
2) Define the freq response of the CT system. **[Nov '11]**

The **response** of a CT system to a **complex sinusoidal** signal gives the **freq response** of the CT system.

i.e., $y(t) = Ae^{j\Omega t} * h(t)$; $DTFT[y(t)] = [Ae^{j\Omega t} * h(t)]$ i.e., $Y(j\Omega) = X(j\Omega)H(j\Omega)$

Here, H(jΩ) is the **freq response of the CT system.**

3) List & draw the basic elements for the block diagram rep'n of CT system. [Nov '12]



4) Check the causality of the system with impulse response h(t) = e^{-t}u(t). **[Nov '12]**

For an **LTI-CT system** to be **causal**, **h(t)** = **0** for t < 0.

Here, **h(t) = e^{-t}u(t) = 0** for **t < 0.** Hence**, Causal** system.

5) What are the 3 elementary operations in block diagram rep'n of CT s/m? [Nov '13]

Adder, Delay, multiplier

6) Check whether the causal system with $H(s) = \frac{1}{s-2}$ **is stable.** **[Nov '13]**

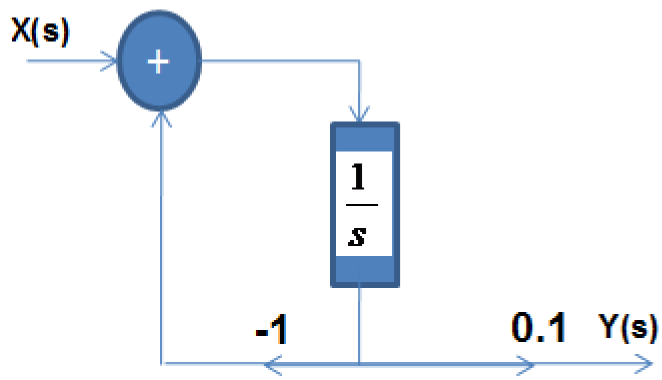
Here, h(t) = e^{2t} u(t). Since **h(t) = 0** for t < 0, it is a **Stable** system.

7) Draw the block diagram of the LTI system $\frac{d}{dt}y(t) + y(t) = 0.1x(t).$ **[Nov '14](R13)**

Taking LT,

sY(s)+Y(s) = 0.1X(s)

i.e., H(s) = $\frac{Y(s)}{X(s)} = \frac{0.1}{s+1} = \frac{\frac{0.1}{s}}{1+\frac{1}{s}}$



8) List the properties of Convolution Integral. **[Nov '14]**

(i) Commutative Property : $x_1(t) * x_2(t) = x_2(t) * x_1(t)$
(ii) Associative Property : $x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$
(iii) Distributive Property : $x_1(t) * [x_2(t) + x_3(t)] = [x_1(t) * x_2(t)] + [x_1(t) * x_3(t)]$

9) State the significance of Impulse Response. **[Nov '14]**

An impulse response refers to the reaction of any dynamic system in response to some external change. In both cases, the impulse response describes the reaction of the system as a function of time.

Since the impulse function contains all frequencies, the impulse response defines the response of a linear time-invariant system for all frequencies.

10) What is $u(t-2) * \delta(t-1)$ **? where** $*$ **represents Convolution?** **[Nov '15](R13)**

$$u(t-2) * \delta(t-1) = \int_{2}^{\infty} \delta(t-1)dt = 0$$

11) Given the differential equation representation of a system

$\frac{d^2}{dt^2}y(t)+2\frac{d}{dt}y(t)-3y(t)=2x(t)$. **Find the Freq response H(jΩ).** **[Nov ’15] (R13)**

Taking **CTFT**, $(j\Omega)^2$ Y(jΩ)+2 jΩ Y(jΩ)-3Y(jΩ) = X(jΩ)

$$H(j\Omega)=\frac{Y(j\Omega)}{X(j\Omega)}=\frac{2}{(j\Omega)^2+2j\Omega-3}$$

12) State the condition for LTI system to be causal & stable. **[Nov ’15]**

h(n)=0, n<0, depends on past and present values not future

13) Differentiate between natural response & forced response. **[Nov ’15]**

Natural Response is the system’s response to initial conditions with all external forces set to zero. Voltage Sources as Short Circuits & Current Sources as Open Circuits.

Forced Response is the system’s response to an external stimulus with zero initial conditions. In circuits, this would be the response of the circuit to external voltage current source forcing function.

14) What is meant by impulse response of any system? **[May ’11]**

If the input is impulse[i.e.,δ(t) or δ(n)], then the Output or Response is **Impulse Response.** i.e., y(t)=T[δ(t)] for CT s/m & y(n)=T[δ(n)] for DT s/m.

15) Define Convolution integral of CT systems. **[May ’11]**

The **Convolution integral** gives the output or response of a CT system which is the **Convolution** of the **input sequence[x(t)] & Impulse response[h(t)]** sequence.

$y(t)=x(t)*h(t)$ $\quad$ $y(t)=\int x(\tau)h(t-\tau)d\tau$

16) Find the differential eqn relating the i/p & o/p of a CT s/m rep'd by

$$H(j\Omega) = \frac{4}{(j\Omega)^2 + 8j\Omega + 4}$$ **[May '14]**

Given $H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \frac{4}{(j\Omega)^2 + 8j\Omega + 4}$

$$(j\Omega)^2 Y(j\Omega) + 8j\Omega Y(j\Omega) + 4Y(j\Omega) = 4X(j\Omega)$$

Taking Inv DTFT, $\frac{d^2}{dt^2}y(t) + 8\frac{d}{dt}y(t) + 4y(t) = 4x(t)$ is the **differential eqn.**

17) Find whether the following system whose impulse response given is causal & stable. $h(t) = e^{-2t}u(t-1)$ **[May '16](R13)**

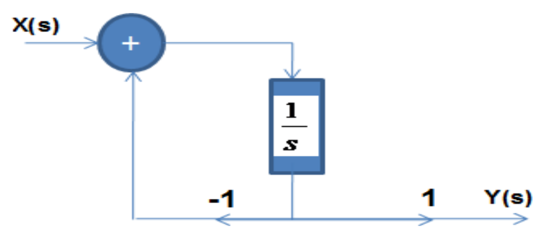
(i) Since h(t) = 0 for t<0, the given system is Causal.

(ii) For a Stable system, $\int |h(t)| dt < \infty$ = $\int_1^\infty e^{-2t} dt = \frac{e^{-2}}{2} < \infty$ $\therefore$ Stable System.

18) Realize the block diagram representing the system $H(s) = \frac{s}{s+1}$. **[May '16](R13)**

Divide Numerator & Denominator by s,

$$H(s) = \frac{1}{1 + \frac{1}{s}}$$



UNIT III CONTINUOUS TIME LTI SYSTEM

PART - B

Q.1: Find the convolution by graphical method

$$x(t) = \begin{matrix} 1 \ for \ \ 0 \le t \le 2 \\ 0 \ \ otherwise \end{matrix} \quad ; \ h(t) = \begin{matrix} 1 \ for \ \ 0 \le t \le 3 \\ 0 \ \ otherwise \end{matrix}$$

Solution:

$$In\ general \quad x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$Similarly\ h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Replacing t by τ in $x(t)$and $h(t)$

$$x(\tau) = \begin{cases} 1 \ for \ \ 0 \le \tau \le 2 \\ 0 \ \ otherwise \end{cases} \quad ; \ h(\tau) = \begin{matrix} 1 \ for \ \ 0 \le \tau \le 3 \\ 0 \ \ otherwise \end{matrix}$$

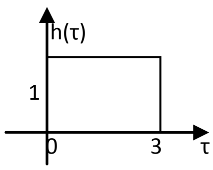


Fig 3.21

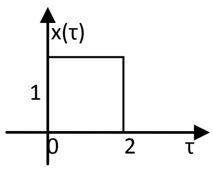


Fig 3.22

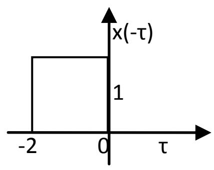


Fig 3.23

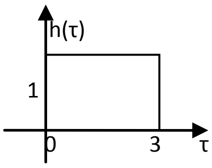


Fig 3.21

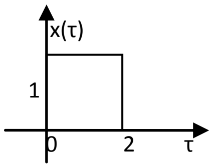


Fig 3.22

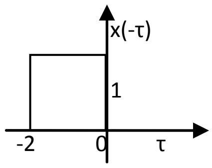


Fig 3.23

Case (i) $\boldsymbol{t} < 0$

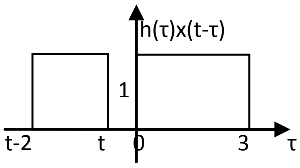


Fig 3.24

Since overlap is absent between $h(\tau)$ and $x(-\tau + t)$

$$\therefore y(t) = h(t) * x(t) = 0$$

Case (ii) $\mathbf{0} \le \boldsymbol{t} < 2$

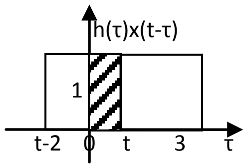


Fig 3.25

Since overlap is present

$$\therefore y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{0}^{t} (1)(1)\, d\tau = [\tau]_0^t = t$$

Case (iii) $\mathbf{2} \le \boldsymbol{t} < 3$

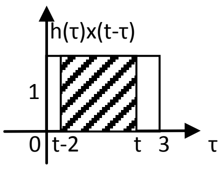


Fig 3.26

Since overlap is present

$$\therefore y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{t-2}^{t} (1)(1)\, d\tau = [\tau]_{t-2}^t = 2$$

Case (iv) $\mathbf{3} \le \boldsymbol{t} < 5$

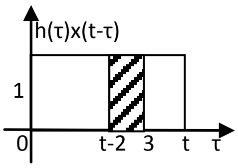


Fig 3.27

Since overlap is present

$$\therefore y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{t-2}^{3} (1)(1)\, d\tau = [\tau]_{t-2}^3$$

$$= 5 - t$$

Case (v) $\boldsymbol{t} > 5$

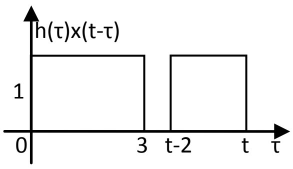


Fig 3.28

Since overlap is absent

$$\therefore y(t) = h(t) * x(t) = 0$$

$$\boxed{\therefore y(t) = h(t) * x(t) = \begin{cases} 0 & for \quad t < 0 \\ t & for \ 0 \le t < 2 \\ 2 & for \ 2 \le t < 3 \\ 5 - t & for \ 3 \le t < 5 \\ 0 & for \quad t \ge 5 \end{cases}}$$

Q. 2: Find impulse response of the following equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$$

Solution:

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = x(t)$$

Assume all the initial conditions are zero

Applying Laplace transform of the given equation

$$S^2Y(S) + 5SY(S) + 6Y(S) = X(S)$$

$$Y(S)(S^2 + 5S + 6) = X(S)$$

Transfer function $H(S) = \frac{Y(S}{X(S)} = \frac{1}{(S^2+5S+6)}$

$$H(S) = Y(S) = \frac{1}{S^2 + 5S + 6} \qquad (\because For\ impulse\ input\ x(t) = \delta(t) => X(S) = 1)$$

$$H(S) = \frac{1}{(S+3)(S+2)} = \frac{A}{S+3} + \frac{B}{S+2}$$

$$1 = A(S+2) + B(S+3)$$

at $S = -3$ | $S = -2$

$A = -1$ | $B = 1$

$$\therefore H(S) = -\frac{1}{S+3} + \frac{1}{S+2}$$

Applying Inverse Laplace transform

$$\boldsymbol{h(t) = -e^{-3t}u(t) + e^{-2t}u(t)}$$

Q.3: Using Laplace transform solve differential equation

$$\frac{d^2y(t)}{dt^2} + y(t) = x(t)$$

Where $y'(0) = 2; y(0) = 1; input\ x(t) = Cos2t$

Solution:

$$\frac{d^2y(t)}{dt^2} + y(t) = x(t)$$

Applying Laplace transform

$$S^2Y(S) - Sy(0) - y'(0) + Y(S) = X(S)$$

$$S^2Y(S) - S - 2 + Y(S) = \frac{S}{S^2 + 4}$$

$$\therefore Y(S)(S^2 + 1) = \frac{S}{S^2 + 4} + S + 2$$

$$Y(S) = \frac{S}{(S^2+4)(S^2+1)} + \frac{S}{(S^2+1)} + \frac{2}{(S^2+1)}$$

$$Let\ \frac{S}{(S^2+4)(S^2+1)} = \frac{AS+B}{(S^2+4)} + \frac{CS+D}{(S^2+1)}$$

$$S = (AS+B)(S^2+1) + (CS+D)(S^2+4)$$

$$S = AS^3 + BS^2 + AS + B + CS^3 + DS^2 + 4CS + 4D$$

Comparing constant term

$0 = B + 4D$

$B = -4D$ ... (7)

Comparing coeff of S^3

$0 = A + C$

$A = -C$ ... (8)

Comparing coeff of S^2

$0 = B + D$ ... (9)

Comparing coeff of S

$1 = A + 4C$ ... (10)

Substitute $eq\ \langle 8\rangle$ in $eq\ \langle 10\rangle$ and $eq\ \langle 7\rangle$ in $eq\ \langle 9\rangle$

$\boldsymbol{C = \frac{1}{3}, D = 0}$

Substitute value of C and D in $eq\ \langle 8\rangle$ and $eq\ \langle 7\rangle$

$\boldsymbol{A = -\frac{1}{3}, B = 0}$

$$\therefore \frac{S}{(S^2+4)(S^2+1)} = \frac{-\frac{1}{3}S}{(S^2+4)} + \frac{\frac{1}{3}S}{(S^2+1)}$$

$$Y(S) = \frac{-\frac{1}{3}S}{(S^2+4)} + \frac{\frac{1}{3}S}{(S^2+1)} + \frac{S}{(S^2+1)} + \frac{2}{(S^2+1)}$$

$$Y(S) = \frac{-\frac{1}{3}S}{(S^2+4)} + \frac{\frac{4}{3}S}{(S^2+1)} + \frac{2}{(S^2+1)}$$

Taking Inverse Laplace transform

$$\boldsymbol{y(t) = -\frac{1}{3}cos2t\,u(t) + \frac{4}{3}cost\,u(t) + 2sint\,u(t)}$$

Q.4: Find step response of the circuit shown in Fig 3.30

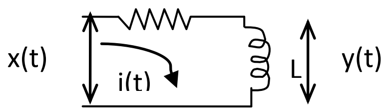


Fig 3.30

Solution:

Applying KVL to the circuit shown in Fig 3.30

$$x(t) = Ri(t) + L\frac{di(t)}{dt} \qquad\qquad y(t) = L\frac{di(t)}{dt}$$

Applying Laplace transform

$$X(S) = RI(S) + LSI(S) \qquad\qquad Y(S) = LSI(S)$$

$$X(S) = [R + LS]I(S)$$

$$I(S) = \frac{X(S)}{[R + LS]}$$

$$Y(S) = LS\frac{X(S)}{[R + LS]}$$

For Step response $x(t) = u(t) \;=>\; X(S) = \frac{1}{S}$

$$Y(S) = LS\frac{\frac{1}{S}}{R + LS} = \frac{L}{LS + R} = \frac{1}{S + \frac{R}{L}}$$

Applying Inverse Laplace transform

$$\boldsymbol{y(t) = e^{-\frac{R}{L}t}u(t)}$$

Q.5: Solve the differential equation using Fourier transform

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

(i) Find the impulse response of the system

(ii) What is the response of the system if $x(t) = te^{-2t}u(t)$

Solution:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

Applying Fourier transform

$$(j\Omega)^2\mathrm{Y}(j\Omega) + 6j\Omega Y(j\Omega) + 8Y(j\Omega) = 2\mathrm{X}(j\Omega)$$

$$\mathrm{Y}(j\Omega)[(j\Omega)^2 + 6j\Omega + 8] = 2\mathrm{X}(j\Omega)$$

$$\mathrm{H}(j\Omega) = \frac{\mathrm{Y}(j\Omega)}{\mathrm{X}(j\Omega)} = \frac{2}{[(j\Omega)^2 + 6j\Omega + 8]}$$

(i) Impulse response $x(t) = \delta(t) \;=>\; X(j\Omega) = 1$

$$\therefore \mathrm{H}(j\Omega) = \mathrm{Y}(j\Omega) = \frac{2}{[(j\Omega)^2 + 6j\Omega + 8]} = \frac{A}{j\Omega + 4} + \frac{B}{j\Omega + 2}$$

$$2 = A(j\Omega + 2) + B(j\Omega + 4)$$

at $j\Omega = -4$ | $j\Omega = -2$

$A = -1$ | $B = 1$

$$\mathrm{H}(j\Omega) = \frac{-1}{j\Omega + 4} + \frac{1}{j\Omega + 2}$$

Applying Inverse Fourier Transform

$$\boldsymbol{h(t) = -e^{-4t}u(t) + e^{-2t}u(t)}$$

(ii) $x(t) = te^{-2t}u(t)$

$$X(j\Omega) = \frac{1}{(j\Omega + 2)^2}$$

$$\frac{\mathrm{Y}(j\Omega)}{\mathrm{X}(j\Omega)} = \frac{2}{[(j\Omega)^2 + 6j\Omega + 8]}$$

$$\therefore \mathrm{Y}(j\Omega) = \frac{2}{[(j\Omega)^2 + 6j\Omega + 8]}\cdot\frac{1}{(j\Omega + 2)^2} = \frac{2}{(j\Omega + 4)(j\Omega + 2)^3}$$

$$= \frac{A}{j\Omega + 4} + \frac{B}{j\Omega + 2} + \frac{C}{(j\Omega + 2)^2} + \frac{D}{(j\Omega + 2)^3}$$

at $j\Omega = -4$

$$A = \frac{2}{(j\Omega+4)(j\Omega+2)^3}(j\Omega+4)\bigg|_{j\Omega=-4}$$

$$A = -\frac{1}{4}$$

$j\Omega = -2$

$$B = \frac{1}{2!}\frac{d^2\left[\frac{2}{(j\Omega+4)(j\Omega+2)^3}(j\Omega+2)^3\right]}{d(j\Omega)^2}\Bigg|_{j\Omega=-2}$$

$$B = \frac{1}{4}$$

$j\Omega = -2$

$$C = \frac{d\left[\frac{2}{(j\Omega+4)(j\Omega+2)^3}(j\Omega+2)^3\right]}{d(j\Omega)}\Bigg|_{j\Omega=-2}$$

$$C = -\frac{1}{2}$$

$j\Omega = -2$

$$D = \frac{2}{(j\Omega+4)(j\Omega+2)^3}(j\Omega+2)^3\bigg|_{j\Omega=-2}$$

$$D = 1$$

$$\therefore \mathrm{Y}(j\Omega) = \frac{-\frac{1}{4}}{j\Omega+4} + \frac{\frac{1}{4}}{j\Omega+2} + \frac{-\frac{1}{2}}{(j\Omega+2)^2} + \frac{1}{(j\Omega+2)^3}$$

Applying Inverse Fourier Transform

$$\boldsymbol{y(t) = -\frac{1}{4}e^{-4t}u(t) + \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}e^{-2t}tu(t) + \frac{1}{2}e^{-2t}t^2u(t)}$$

Q.6: Find the direct form II structure of

$$H(S) = \frac{5S^3 - 4S^2 + 11S - 2}{\left(S-\frac{1}{4}\right)\left(S^2 - S + \frac{1}{2}\right)}$$

Solution:

$$H(S) = \frac{5S^3 - 4S^2 + 11S - 2}{\left(S-\frac{1}{4}\right)\left(S^2 - S + \frac{1}{2}\right)} = \frac{5S^3 - 4S^2 + 11S - 2}{S^3 - \frac{S^2}{4} - S^2 + \frac{S}{4} + \frac{S}{2} - \frac{1}{8}}$$

$$H(S) = \frac{5S^3 - 4S^2 + 11S - 2}{S^3 - \frac{5S^2}{4} + \frac{3S}{4} - \frac{1}{8}} = \frac{5 - \frac{4}{S} + \frac{11}{S^2} - \frac{2}{S^3}}{1 - \frac{5}{4S} + \frac{3}{4S^2} - \frac{1}{8S^3}}$$

Direct form II structure

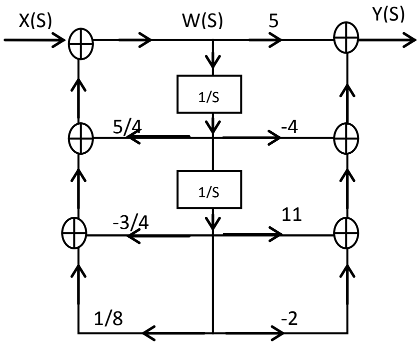


Fig 3.39

Q.7: Realize the system with following differential equation in direct form I

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 7y(t) = 2\frac{d^2x(t)}{dt^2} + 0.4\frac{dx(t)}{dt} + 0.5x(t)$$

Solution:

$$\frac{d^3y(t)}{dt^3}+3\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+7y(t)=2\frac{d^2x(t)}{dt^2}+0.4\frac{dx(t)}{dt}+0.5x(t)$$

Taking Laplace transform

$$S^3Y(S)+3S^2Y(S)+5SY(S)+7Y(S)=2S^2X(S)+0.4SX(S)+0.5X(S)$$

Dividing both the side by S^3

$$Y(S)+\frac{3}{S}Y(S)+\frac{5}{S^2}Y(S)+\frac{7}{S^3}Y(S)=\frac{2}{S}X(S)+\frac{0.4}{S^2}X(S)+\frac{0.5}{S^3}X(S)$$

$$Y(S)=\frac{2}{S}X(S)+\frac{0.4}{S^2}X(S)+\frac{0.5}{S^3}X(S)-\frac{3}{S}Y(S)-\frac{5}{S^2}Y(S)-\frac{7}{S^3}Y(S)$$

Direct form I structure

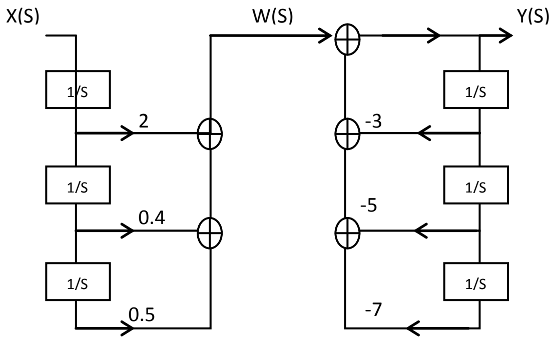


Fig 3.42

Q.8: Realize the system with transfer function in cascade form

$$H(S)=\frac{4(S^2+4S+3)}{S^3+6.5S^2+11S+4}$$

Solution:

$$H(S)=\frac{4(S^2+4S+3)}{S^3+6.5S^2+11S+4}=\frac{4(S+1)(S+3)}{(S+0.5)(S+2)(S+4)}=\frac{4}{S+0.5}\cdot\frac{S+1}{S+2}\cdot\frac{S+3}{S+4}$$

$$H_1(S)H_2(S)H_3(S)=\frac{4}{S+0.5}\cdot\frac{S+1}{S+2}\cdot\frac{S+3}{S+4}$$

$$H_1(S)=\frac{4}{S+0.5}=\frac{4/S}{1+0.5/S}$$

$$\frac{Y_1(S)}{W_1(S)}=\frac{4}{S}$$

$$\frac{W_1(S)}{X_1(S)}=\frac{1}{1+\frac{0.5}{S}}$$

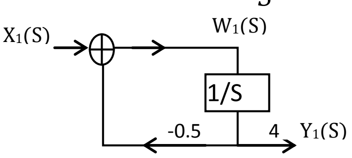


Fig 3.54

$$H_2(S)=\frac{S+1}{S+2}=\frac{1+1/S}{1+2/S}$$

$$\frac{Y_2(S)}{W_2(S)}=1+1/S$$

$$\frac{W_2(S)}{X_2(S)}=\frac{1}{1+2/S}$$

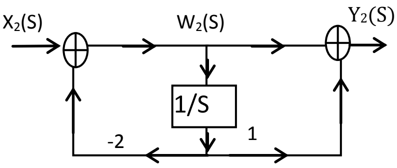


Fig 3.55

$$H_3(S)=\frac{S+3}{S+4}=\frac{1+3/S}{1+4/S}$$

$$\frac{Y_3(S)}{W_3(S)}=1+3/S$$

$$\frac{W_3(S)}{X_3(S)}=\frac{1}{1+4/S}$$

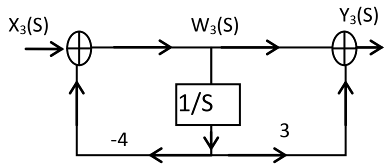


Fig 3.56

Cascade form:

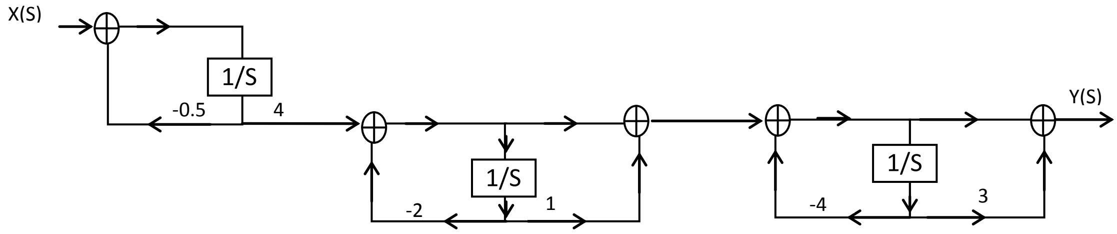


Fig 3.57

Q.9: Realize the following system in parallel form

$$H(S) = \frac{S(S+2)}{(S+1)(S+3)(S+4)}$$

Solution:

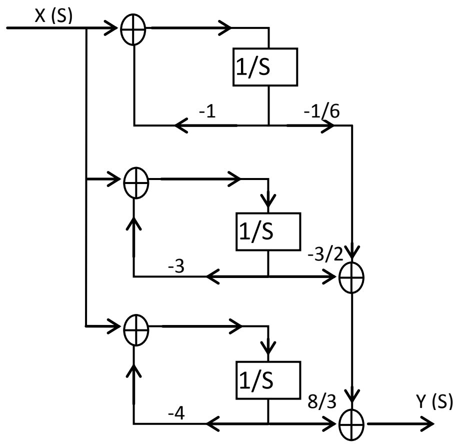
$$H(S) = \frac{S(S+2)}{(S+1)(S+3)(S+4)} = \frac{A}{S+1} + \frac{B}{S+3} + \frac{C}{S+4}$$

$$S(S+2) = A(S+3)(S+4) + B(S+1)(S+4) + C(S+1)(S+3)$$

Let S= -1	Let S= -3	Let S= -4
$-1(1) = A(2)(3)$	$-3(-1) = B(-2)(1)$	$-4(-2) = C(-3)(-1)$
$\boldsymbol{A = -\frac{1}{6}}$	$\boldsymbol{B = -\frac{3}{2}}$	$\boldsymbol{C = \frac{8}{3}}$

$$\therefore H(S) = \frac{-\frac{1}{6}}{S+1} + \frac{-\frac{3}{2}}{S+3} + \frac{\frac{8}{3}}{S+4}$$

Parallel form structure



UNIT IV

PART A

UNIT IV : ANALYSIS OF DISCRETE TIME SIGNALS

1) Define Sampling theorem. **[May '11]**

A band limited x(t) with X(jΩ)=0 for $|\Omega| > \Omega_m$ is uniquely determined from its samples x(nT), if the sampling freq $\mathbf{f_s \geq 2f_m}$ where, f_m : highest freq present in the signal.

2) What is ROC in Z-transforms? **[May '11]**

Refer **Q.No.10 ; Nov 2014**

3) Prove the Time-Shifting property of DTFT. **[May '12]**

$$DTFT\,[x(n \pm k)] = e^{\pm j\omega k} X\,(e^{j\omega})$$

Proof: $DTFT[x(n-k)] = \sum_n x(n-k)e^{-j\omega n}$ **Let** m=n-k i.e.,n=m+k

$$\therefore DTFT[x(n-k)] = \sum_m x(m)e^{-j\omega(m+k)} = e^{-j\omega k}\sum_n x(m)e^{-j\omega m} \quad = e^{-j\omega k} X\,(e^{j\omega})$$

4) List any 2 properties of ROC in Z-domain. **[Nov '11]**

(i) The ROC does not contain any poles.
(ii) The ROC must be a connected region.

5) Define DTFT & Inverse DTFT. **[Nov '12]**

$$DTFT[x(n)] = X\,(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

Inv DTFT[X($e^{j\omega}$)] = $\frac{1}{2\pi}\int X\,(e^{j\omega})e^{j\omega n}$

6) State the Convolution property of Z-transform. **[Nov '12]**

$z[x(n) * h(n)] = X\,(z)H\,(z)$ **…i.e.,** The **Z-transform** of the **Convolution** in the **time domain** is equal to the **Product** of their **Z-transforms** in the **freq domain**.

7) What is Aliasing? **[Nov ’13]**

In the frequency spectrum of sampling theorem, if $\frac{\pi}{T} < \Omega_m$, the successive freq replicas will overlap & as a result, the frequency spectrum X(jΩ) will not be recovered from the frequency spectrum of $TX_s(j\Omega)$. The superimposition of high freq components on the low freq is known as freq Aliasing.

8) Define unilateral & bilateral Z-Transform. **[Nov ’13]**

Unilateral Z-Transform : $Z[x(n)] = X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$ **(Causal Sequence)**

$Z[x(n)] = X(z) = \sum_{n=-\infty}^{-1} x(n) z^{-n}$ **(Non-Causal Seq)**

Bilateral Z-Transform : $Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

9) Find $y(n) = x(n-1) * \delta(n-2)$. **[Nov ’14](R13)**

W.K.T, $y(n) = x(n) * h(n)$ **i.e.,** $y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$

$\therefore y(n) = x(n-1) * \delta(n-2) = x(n-1)$ at **n=2.**

10)Find the DTFT of $x(n) = \delta(n) + \delta(n-1)$. **[Nov ’14] (R13)**

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n} [\delta(n) + \delta(n-1)] = 1 + e^{-j\omega}$$

11)State & Prove the time-folding property of z-transform. **[Nov ’14] (R13)**

$$Z[x(n-k)] = \sum_{n} x(n-k) z^{-n} = \sum_{n} x(m) z^{-(m+k)} = z^{-k} \sum_{m} x(m) z^{-m}$$

$$= z^{-k} X(z)$$

12) Write a note on ROC. **[Nov ’14]**

The values of ‘z’ for which the summation $\sum_{k=-\infty}^{\infty} x(n)z^{-n}$ converges is called the Region of Convergence(ROC).

13) State the need for Sampling. **[Nov ’15](R13)**

Sampling is a process of converting a continuous time signal to discrete time signal.
Sampling Theorem ; $f_s \geq 2f_m$. where, f_s :Sampling Frequency & f_m :Maximum Signal Freq.

14) Find the z-transform & its associated ROC for $x(n) = \{1,-1,2,3,4\}$? [Nov ’15] (R13)

$$X(z) = \sum_{n=-3}^{1} x(n)z^{-n} = x(-3)z^{-(-3)} + x(-2)z^{-(-2)} + \ldots x(1)z^{-1} = 1.z^{3} - z^{2} + 2z + 3 + 4z^{-1}$$

ROC: Entire Z-plane except at **z = 0**.

15) Define Z-transform. **[Nov ’15]**

$$X(z) \sum = x(n)z^{-n}$$

16) State the relation between DTFT & Z-transform. **[Nov ’15]**

$DTFT[x(n)] = X(e^{j\omega}) = X(z)$ at $z=e^{j\omega}$

i.e., $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$ at **$z = e^{j\omega}$ =** $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$

17)State the Final value theorem. **[May ’12]**

Final value theorem: $x(\infty) = \lim_{z \to 1} (z-1)X(z)$

18) Find the final value of the given signal $X(z) = \frac{1}{1+2z^{-1}+3z^{-2}}$ **[May ’16](R13)**

Given $X(z) = \frac{1}{(1+2z^{-1})(1+z^{-1})}$ (or) $X(z) = \frac{z^{2}}{(z+2)(z+1)}$.

Here, (z-1)X(z) has a pole on the unit circle.

$$\therefore \lim (z-1)\frac{z^{2}}{(z+2)(z+1)} = x(\infty) = 0$$

UNIT 4 DICSRETE TIME SIGNAL ANALYSIS

PART B

Q.1: Find Fourier transform of the following

i) $x(n) = \delta(n)$

ii) $x(n) = u(n)$

iii) $x(n) = a^n u(n)$

Solution:

i) $x(n) = \delta(n)$

$$X(e^{j\omega}) = F\{\delta(n)\} = \sum_{n=-\infty}^{\infty} \delta(n)\, e^{-j\omega n} = e^0 = 1$$

$$\delta(n) = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

ii) $x(n) = u(n)$

$$X(e^{j\omega}) = F\{u(n)\} = \sum_{n=-\infty}^{\infty} u(n)\, e^{-j\omega n} = \sum_{n=0}^{\infty} e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-2j\omega} + \cdots \infty = \frac{1}{1 - e^{-j\omega}}$$

$$u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

$$1 + x + x^2 + \cdots = \frac{1}{1 - x}$$

iii) $x(n) = a^n u(n)$

It is **Right handed exponential signal**

$$X(e^{j\omega}) = F\{a^n u(n)\} = \sum_{n=-\infty}^{\infty} a^n u(n) e^{-j\omega n} = \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= 1 + ae^{-j\omega} + (ae^{-j\omega})^2 + \cdots + \infty = \frac{1}{1 - ae^{-j\omega}}$$

Q.2: Obtain DTFT of rectangular pulse

$$x(n) = \begin{cases} A, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$

Solution:

$$X(e^{j\omega}) = \sum_{n=0}^{L-1} Ae^{-j\omega n} = A\left[\frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}\right] \qquad \sum_{n=0}^{N-1} x^n = \frac{1 - x^N}{1 - x}$$

$$X(e^{j\omega}) = A\left[\frac{\left(e^{\frac{j\omega L}{2}} - e^{\frac{-j\omega L}{2}}\right)e^{\frac{-j\omega L}{2}}}{\left(e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}}\right)e^{\frac{-j\omega}{2}}}\right] = A\left[\frac{2j\sin\frac{\omega L}{2}}{2j\sin\frac{\omega}{2}}\right]e^{-\frac{j\omega\,(L-1)}{2}} = Ae^{-\frac{j\omega\,(L-1)}{2}}\left[\frac{\sin\frac{\omega L}{2}}{\sin\frac{\omega}{2}}\right]$$

Q.3 Find DTFT of $x(n) = \sin(\mathrm{n}\,\theta)\mathrm{u(n)}$ Solution:

$$x(n) = \sin(\mathrm{n}\,\theta)\mathrm{u(n)}$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} \sin(\mathrm{n}\,\theta)e^{-j\omega n} = \sum_{n=0}^{\infty}\left(\frac{\mathrm{e}^{\mathrm{j}\theta\mathrm{n}} - \mathrm{e}^{-\mathrm{j}\theta\mathrm{n}}}{2\mathrm{j}}\right)e^{-\mathrm{j}\omega n} = \frac{1}{2j}\left(\sum_{n=0}^{\infty}\mathrm{e}^{\mathrm{j}(\theta-\omega)\mathrm{n}} - \sum_{n=0}^{\infty}\mathrm{e}^{-\mathrm{j}(\theta+\omega)\mathrm{n}}\right)$$

$$= \frac{1}{2j}\left(\frac{1}{1 - \mathrm{e}^{\mathrm{j}(\theta-\omega)}} - \frac{1}{1 - \mathrm{e}^{-\mathrm{j}(\theta+\omega)}}\right) = \frac{1}{2j}\left(\frac{1 - \mathrm{e}^{-\mathrm{j}(\theta+\omega)} - 1 + \mathrm{e}^{\mathrm{j}(\theta-\omega)}}{1 - 2\mathrm{e}^{-\mathrm{j}\omega}\cos\theta + e^{-2j\omega}}\right)$$

$$= \frac{1}{2j}\left(\frac{2\mathrm{j}\mathrm{e}^{-\mathrm{j}\omega}\sin\theta}{1 - 2\mathrm{e}^{-\mathrm{j}\omega}\cos\theta + e^{-2j\omega}}\right) = \frac{\mathrm{e}^{-\mathrm{j}\omega}\sin\theta}{1 - 2\mathrm{e}^{-\mathrm{j}\omega}\cos\theta + e^{-2j\omega}}$$

4.4 Z-Transform

The Z-transform of discrete time signal $x(n)$ is defined as

$$Z[x(n)] = X(Z) = \sum_{n=-\infty}^{\infty} x(n)Z^{-n}$$

4.4.2 Inverse Z-transform

The inverse Z-transform of $X(Z)$ is defined as

$$x(n) = Z^{-1}[X(Z)] = \frac{1}{2\pi j}\oint_C X(Z)\,Z^{n-1}dZ$$

Q.4: Find Z-transform of the following

i) $x(n) = \delta(n)$

ii) $x(n) = u(n)$

iii) $x(n) = -a^n u(-n-1)\ \ and\ find\ ROC$

i) $x(n) = \delta(n)$

$$\delta(n) = \begin{matrix} 1 \; for \; n = 0 \\ 0 \; for \; n \neq 0 \end{matrix}$$

$$Z[\delta(n)] = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} = Z^{-0} = 1$$

$$\therefore Z[\delta(n)] = 1$$

ii) $x(n) = u(n)$

$$Z[u(n)] = \sum_{n=-\infty}^{\infty} u(n) z^{-n} = \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots\cdots = 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\cdots$$

$$= \frac{1}{1 - \frac{1}{z}} = \frac{1}{1 - z^{-1}} = \frac{Z}{Z - 1}$$

The above series convergence if $|Z^{-1}| < 1$ i.e ROC is $|Z| > 1$

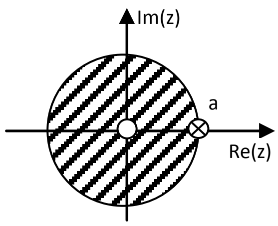
iii) $x(n) = -a^n u(-n-1)$

$$X(Z) = \sum_{n=\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} -a^n u(-n-1) z^{-n} = -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$X(Z) = -\sum_{n=1}^{\infty} a^{-n} z^n = -[a^{-1}Z + (a^{-1}Z)^2 + (a^{-1}Z)^3 + \cdots] = -a^{-1}Z[1 + a^{-1}Z + (a^{-1}Z)^2 + \cdots]$$

$$= -\left[\frac{a^{-1}z}{1 - a^{-1}z}\right] = \frac{-a^{-1}}{-a^{-1}z}\left[\frac{z}{1 - az^{-1}}\right] = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}$$

ROC: $|a^{-1}Z| < 1 \Rightarrow |Z| < |a|$



Q.5: Obtain Inverse Z-Transform of

$$X(z) = \frac{1}{1 - 0.6Z^{-1} + 0.08Z^{-2}} \qquad for \;\; i)\; |Z| > 0.4 \;\; ii)\; |Z| < 0.2 \;\; iii) 0.2 < |Z| < 0.4$$

Solution:

$$X(z) = \frac{1}{1 - 0.6Z^{-1} + 0.08Z^{-2}} = \frac{Z^2}{Z^2 - 0.6Z + 0.08}$$

$$\frac{X(Z)}{Z} = \frac{Z}{(Z - 0.2)(Z - 0.4)} = \frac{A}{Z - 0.2} + \frac{B}{Z - 0.4}$$

$$A = \frac{Z}{(Z - 0.2)(Z - 0.4)}(Z - 0.2)\Big|_{Z=0.2} = -1 \qquad \Bigg| \qquad B = \frac{Z}{(Z - 0.2)(Z - 0.4)}(Z - 0.4)\Big|_{Z=0.4} = 2$$

$$\therefore \frac{X(Z)}{Z} = \frac{-1}{Z - 0.2} + \frac{2}{Z - 0.4} \Rightarrow X(Z) = \frac{-Z}{Z - 0.2} + \frac{2Z}{Z - 0.4}$$

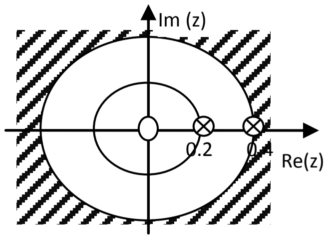
Applying inverse Z-transform

$$x(n) = -(0.2)^n u(n) + 2(0.4)^n u(n)$$

ROC: $|Z| > 0.4$

ROC lies outside of all poles. So both the terms are causal

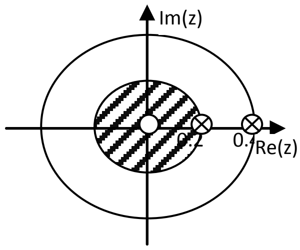
$$\therefore x(n) = -(0.2)^n u(n) + 2(0.4)^n u(n)$$



ROC : $|Z| < 0.2$

ROC lies inside of all poles. So both the terms are non-causal

$$\therefore x(n) = (0.2)^n u(-n-1) - 2(0.4)^n u(-n-1)$$

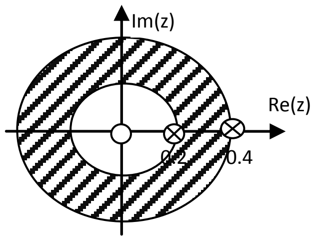


ROC :

$0.5 < |z| < 1$

ROC lies inside of pole Z=0.4 and lies outside of pole Z=0.2. So the term with pole Z=0.4 is non-causal and the term with pole Z=0.2 is causal

$$\therefore x(n) = -(0.2)^n u(n) - 2(0.4)^n u(-n-1)$$



Q.6 Find the inverse Z transform of $X(Z) =$ Solution: $\frac{Z^3}{(Z+1)(Z-1)^2}$ using Cauchy residue method.

$$X(Z) = \frac{Z^3}{(Z+1)(Z-1)^2}$$

$$x(n) = Residue\ of\ X(Z)\ Z^{n-1} at\ pole\ (Z=-1)$$
$$+\ Residue\ of X(Z)\ Z^{n-1} at\ pole\ (Z=1)\ with\ multiplicity\ 2$$

$$x(n) = \left.\frac{Z^3(Z+1)}{(Z+1)(Z-1)^2} Z^{n-1}\right|_{Z=-1} + \left.\frac{d}{dZ}\left[\frac{Z^3(Z-1)^2}{(Z+1)(Z-1)^2}\right] Z^{n-1}\right|_{Z=1}$$

$$= \left(-\frac{1}{4}\right)(-1)^{n-1} + \frac{2(n+2)-1}{4} = \left(-\frac{1}{4}\right)(-1)^{n-1} u(n) + \frac{2n+3}{4} u(n)$$

$x(0) = 1 \quad x(1) = 1 \quad x(2) = 2 \quad x(3) = 2$

$\therefore x(n) = \{1,1,2,2,\dots\}$

UNIT V

PART A

UNIT V : LINEAR TIME INVARIANT-DISCRETE TIME SYSTEMS

1) Find the system response of x(n)=u(n) & h(n)=δ(n)+ δ(n-1). [Nov '11]

$y(n) = x(n) * h(n)$ i.e., $y(n) = u(n) * [\delta(n) + \delta(n-1)]$

$y(n) = [u(n) * \delta(n)] + [u(n) * \delta(n-1)]$ = $u(n) + u(n-1)]$

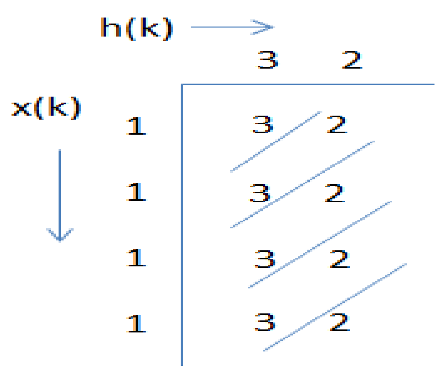
2) List the advantages of state variable representation of a system. [Nov '11]

- It can be applied to non linear system.
- It can be applied to tile invariant systems.
- It can be applied to multiple input multiple output systems.
- Its gives idea about the internal state of the system.

3) Convolve the following 2 sequences. x(n)={1,1,1,1} & h(n)={3,2}. [Nov '12]

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

y(n) = {3,5,5,5,2}



4) A causal LTI system has impulse response h(n), for which the z-transform is $H(z)=\dfrac{1+z^{-1}}{(1-0.5z^{-1})(1-0.25z^{-1})}$. Is the system stable? Explain. **[Nov ’12]**

Converting H(z) into **+ve powers** of ‘z’, $H(z)=\dfrac{z(z+1)}{(z-0.5)(z-0.25)}$

Since the **poles** of H(z) i.e., z=0.5 & z=0.25 lie **inside** the **unit circle**, the system is **stable**.

5) Define Convolution sum with its equation. **[Nov ’13]**

$y(n)=\sum_{k=-\infty}^{\infty} x(k)h(n-k)$. The **Convolution sum** (or) the **Linear Convolution** gives the output or response of a DT system which is the **Convolution** of the **input sequence & Impulse response** sequence.

6) Check whether the system with system function $H(z)=\dfrac{1}{1-\dfrac{1}{2z^{-1}}}+\dfrac{1}{1-2z^{-1}}$ with ROC $|z|<\dfrac{1}{2}$ is causal & stable. **[Nov ’13]**

(i) $H(z)=\dfrac{1}{1-0.5z}+\dfrac{z}{z-2}$. Taking **Inv z-transform**, one of the terms of h(n) = 2^nu(n).

Since **h(n) ≠ 0** for n<0, it is a **Non-Causal** system.

(ii) **One of** the **poles** of H(z) is **outside** the unit circle. Hence, it is an **Unstable** system.

7) Give the impulse response of a linear time invariant system as h(n)=sin(πn), check whether the system is stable or not. **[Nov ’14] (R13)**

For a stable system, $\sum_{n=-\infty}^{\infty}|h(n)|<\infty$ $=\sum_{n=-\infty}^{\infty}|h(n)|$ $=\sum_{n=-\infty}^{\infty}|\sin(\pi n)|$ = 0 < ∞.

∴ The system is **stable**.

8) In terms of ROC, state the condition for an LTI-DT system to be causal & stable. **[Nov '14] (R13)**

For an **LTI-DT** system to be **causal**, h(n) = 0 for n<0.

For an **LTI-DT** system to be **Stable**, $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

9) Write the nth order Difference Equation. **[Nov '14]**

$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$, where, 'N' is called the Order of the system.

10) Write the State Variable equations of a DT-LTI system. **[Nov '14]**

Q(n+1) = A Q(n) + B x(n) **&** y(n) = C Q(n) + D x(n)

11) Distingush between Recursive & Non-recursive systems. **[Nov '15] (R13)**

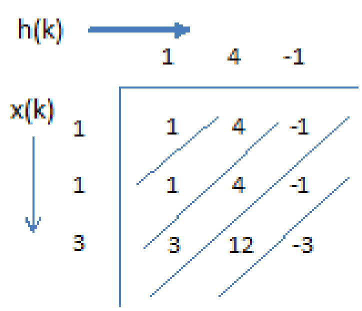
A Recursive system is one in which the output is dependent not only on its present inputs, but also on past outputs.

A Non-Recursive system is one in which the output is dependent only on its present inputs & not on past outputs.

12) Convolve the following signals, x(n) = {1,1,3} & h(n) = {1,-4,1}. **[Nov '15] (R13)**

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

y(n) = {1,5,6,11,-3}



13) List the 4 steps to obtain Convolution. **[Nov '15]**

(i)Folding h(k) ; h(-k)
(ii)Shifting h(k) ; h(n_0-k)
(iii)Multiplication ; x(k) h(n_0-k)
(iv)Summation ; $y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$

14) What is state transition matrix? **[Nov '15]**

The state-transition matrix is used to find the solution to a general state-space representation of a linear system.

State Variable equations : Q(n+1) = A Q(n) + B x(n) : **State Equation**
y(n) = C Q(n) + D x(n) : Output Equation

15)Obtain the Convolution of (i) $x(n) * \delta(n)$ (ii) $x(n) * [h_1(n) + h_2(n)]$ **[May ’11]**

(i) $x(n) * \delta(n) = x(n)$

(ii) $x(n) * [h_1(n) + h_2(n)] = [x(n) * h_1(n)] + [x(n) * h_2(n)]$

16)Write the difference equation for non-recursive system. **[May ’11]**

Non-recursive system is FIR system.

The **difference equation** is $y(n) = \sum^{N-1} b_k x(n-k)$ where, b_k=h(k)=Impulse response samples

17)Determine the Convolution of x(n)={2,-1,3,2} & h(n)={1,-1,1,1}. **[May ’12]**

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

y(n)={2,-3,6,0,0,5,2}

h(k) →

x(k) ↓	2	-1	3	2
1	2	-1	3	2
-1	-2	1	-3	-2
1	2	-1	3	2
1	2	-1	3	2

$n=0$

18) Find the system function for y(n)= 0.5y(n-1)+x(n). **[May ’12]**

Taking z-transform, $Y(z) = 0.5z^{-1}Y(z) + X(z)$

$\therefore$ **System Function,** $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{1}{1-0.5z^{-1}}$

19) Is the DT s/m described by the difference equation y(n)=x(-n) causal? **[May ’13]**

At n= -3, **y(-3) = x(3)** ; Since o/p **depends** on the **future i/p**, it is a **Non-Causal** s/m.

20) If X(ω) is the DTFT of x(n), what is the DTFT of x(-n)? **[May ’13]**

FT[x(-n)] = $\sum_n x(-n)e^{-j\omega n}$ Let **m = -n**.

$$= \sum_m x(m)e^{j\omega m} \quad = \sum_m x(m)e^{-j\omega(-m)}$$ **= X(e^-jω)**

21) Find the overall impulse response when 2 s/ms $h_1(n)$=u(n) & $h_2(n)$= δ(n)+ 2δ(n-1) are in series. **[May ’14]**

Overall I**mpulse response =** $h_1(n) * h_2(n) = u(n) * [\delta(n) + 2\delta(n-1)]$

$$= [u(n) * \delta(n)] + [u(n) * 2\delta(n-1)] = u(n) + 2u(n-1)$$ = {1,3,3,3,3,…..}

22) Using z-transform, check whether the following system is stable. **[May ’14]**

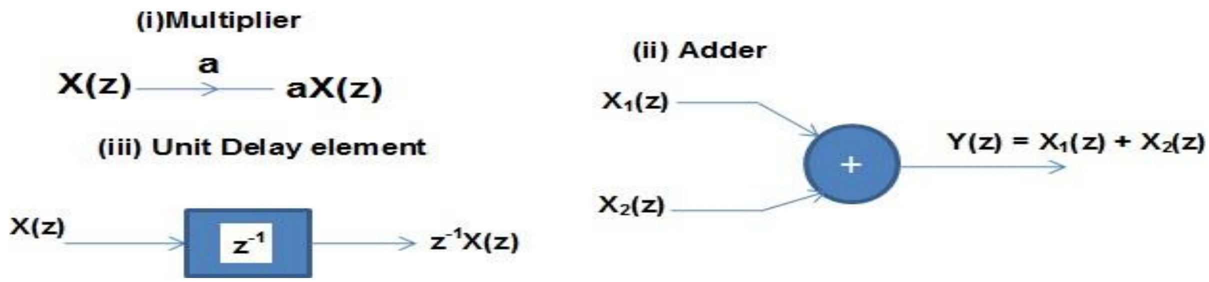
$$H(z) = \frac{z}{z-\frac{1}{2}} + \frac{2z}{z-3}; \qquad ROC: \frac{1}{2} < |z| < 3$$

The poles of H(z) are 0.5 & 3. Since, the pole z=3 is outside the circle, system is said to be Unstable.

23) List the methods used for finding the Inverse Z-transform. **[May ’15] (R13)**

- Long division method(Power series expansion)
- Partial fraction expansion method
- Residue method
- Convolution method

24) Name the basic building blocks used in LTI/DT s/m block diagram. [May ’15](R13)



25) Define stability in LTI system. [May ’15]

A system is said to be stable if a bounded input sequence always produces a bounded output sequence. For an **LTI-DT** system to be **Stable**, $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

26) From Convolution sum, find the step response in terms of h(n). [May ’16](R13)

$s(n) = x(n) * h(n) = u(n) * h(n) \qquad Here,\ x(n) = u(n)$

$s(n) = \sum_{k=-\infty}^{n} h(k)$ For a Causal system, $s(n) = \sum_{k=0}^{n} h(k)$

27) Define the non-recursive system. [May ’16](R13)

A Non-Recursive system is one in which the output is dependent only on its present inputs & not on past outputs.

28) A causal LTI system has impulse response h(n), for which the z-transform is $H(z) = \frac{1+z^{-1}}{(1-0.5z^{-1})(1-0.25z^{-1})}$. Is the system stable? Explain. **[May ’16]**

stable

UNIT V -LTI SYSTEMS

PART -B

Q.1: Determine the frequency response and impulse response $y(n)-\frac{1}{6}y(n-1)-\frac{1}{6}y(n-2)=x(n)$

Applying DTFT

$$Y(e^{j\omega})-\frac{1}{6}e^{-j\omega}Y(e^{j\omega})-\frac{1}{6}e^{-2j\omega}Y(e^{j\omega})=X(e^{j\omega})$$

$$\text{Frequency response } H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-2j\omega}}=\frac{e^{2j\omega}}{e^{2j\omega}-\frac{1}{6}e^{j\omega}-\frac{1}{6}}$$

$$\frac{H(e^{j\omega})}{e^{j\omega}}=\frac{e^{j\omega}}{e^{2j\omega}-\frac{1}{6}e^{j\omega}-\frac{1}{6}}=\frac{A}{e^{j\omega}-\frac{1}{2}}+\frac{B}{e^{j\omega}+\frac{1}{3}}$$

$$e^{j\omega}=A\left(e^{j\omega}+\frac{1}{3}\right)+B\left(e^{j\omega}-\frac{1}{2}\right)$$

At $e^{j\omega}=-\frac{1}{3}$

$-\frac{1}{3}=B\left(-\frac{1}{3}-\frac{1}{2}\right)$, $\therefore B=\frac{2}{5}$

At $e^{j\omega}=\frac{1}{2}$

$\frac{1}{2}=A\left(\frac{1}{2}+\frac{1}{3}\right)$, $\therefore A=\frac{3}{5}$

$$H(e^{j\omega})=\frac{\frac{3}{5}e^{j\omega}}{e^{j\omega}-\frac{1}{2}}+\frac{\frac{2}{5}e^{j\omega}}{e^{j\omega}+\frac{1}{3}}$$

Applying inverse DTFT

$$h(n)=\frac{3}{5}\left(\frac{1}{2}\right)^n u(n)+\frac{2}{5}\left(-\frac{1}{3}\right)^n u(n)$$

Q.2: Find response of system using DTFT

$$h(n)=\left(\frac{1}{2}\right)^n u(n)\ ;\ x(n)=\left(\frac{3}{4}\right)^n u(n)$$

Solution:

$$h(n)=\left(\frac{1}{2}\right)^n u(n)\ ;\ x(n)=\left(\frac{3}{4}\right)^n u(n)$$

Applying DTFT

$$H(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\quad ;\quad X(e^{j\omega})=\frac{1}{1-\frac{3}{4}e^{-j\omega}}$$

$$Y(e^{j\omega})=H(e^{j\omega})X(e^{j\omega})$$

$$Y(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}\cdot\frac{1}{1-\frac{3}{4}e^{-j\omega}}=\frac{e^{j\omega}}{e^{j\omega}-\frac{1}{2}}\cdot\frac{e^{j\omega}}{e^{j\omega}-\frac{3}{4}}$$

$$\frac{Y(e^{j\omega})}{e^{j\omega}}=\frac{e^{j\omega}}{\left(e^{j\omega}-\frac{1}{2}\right)\left(e^{j\omega}-\frac{3}{4}\right)}=\frac{A}{e^{j\omega}-\frac{1}{2}}+\frac{B}{e^{j\omega}-\frac{3}{4}}$$

$$e^{j\omega}=A\left(e^{j\omega}-\frac{3}{4}\right)+B\left(e^{j\omega}-\frac{1}{2}\right)$$

At $e^{j\omega}=\frac{1}{2}$

$\frac{1}{2}=A\left(\frac{1}{2}-\frac{3}{4}\right)$, $\therefore A=-2$

At $e^{j\omega}=\frac{3}{4}$

$\frac{3}{4}=B\left(\frac{3}{4}-\frac{1}{2}\right)$, $\therefore B=3$

$$\frac{Y(e^{j\omega})}{e^{j\omega}}=\frac{-2}{e^{j\omega}-\frac{1}{2}}+\frac{3}{e^{j\omega}-\frac{3}{4}}\ \Rightarrow\ Y(e^{j\omega})=\frac{-2e^{j\omega}}{e^{j\omega}-\frac{1}{2}}+\frac{3e^{j\omega}}{e^{j\omega}-\frac{3}{4}}$$

Applying IDTFT

$$y(n)=-2\left(\frac{1}{2}\right)^n u(n)+3\left(\frac{3}{4}\right)^n u(n)$$

Q.3: Find output response using Z-transform

$$y(n)-\frac{3}{2}y(n-1)+\frac{1}{2}y(n-2)=2x(n)+\frac{3}{2}x(n-1)$$

Where $y(-1)=0 \quad y(-2)=1 \quad x(n)=\left(\frac{1}{4}\right)^n u(n)$

Solution:

$$y(n)-\frac{3}{2}y(n-1)+\frac{1}{2}y(n-2)=2x(n)+\frac{3}{2}x(n-1)$$

Taking Z-transform

$$Y(Z)-\frac{3}{2}[Z^{-1}Y(Z)+y(-1)]+\frac{1}{2}[Z^{-2}Y(Z)+Z^{-1}y(-1)+y(-2)]$$

$$=2X\ (Z)+\frac{3}{2}[Z^{-1}X(Z)+x(-1)]$$

$$Y(Z)-\frac{3}{2}[Z^{-1}Y(Z)]+\frac{1}{2}[Z^{-2}Y(Z)+1]=X(Z)\left[2+\frac{3}{2}Z^{-1}\right] \quad \because y(-1)=0, y(-2)=1$$

$$\textit{Since } x(n) \textit{is causal signal } x(-1)=0$$

$$x(n)=\left(\frac{1}{4}\right)^n u(n)\Rightarrow X(Z)=\frac{1}{1-\frac{1}{4}Z^{-1}}=\frac{Z}{Z-\frac{1}{4}}$$

$$\therefore Y(Z)\left[1-\frac{3}{2}Z^{-1}+\frac{1}{2}Z^{-2}\right]=\frac{Z}{Z-\frac{1}{4}}\left[2+\frac{3}{2}Z^{-1}\right]-\frac{1}{2}$$

$$Y(Z)=\frac{Z}{Z-\frac{1}{4}}\frac{\left(2+\frac{3}{2}Z^{-1}\right)}{\left(1-\frac{3}{2}Z^{-1}+\frac{1}{2}Z^{-2}\right)}-\frac{\frac{1}{2}}{1-\frac{3}{2}Z^{-1}+\frac{1}{2}Z^{-2}}$$

$$Y(Z)=\frac{Z}{Z-\frac{1}{4}}\left(\frac{2Z^2+\frac{3}{2}Z}{Z^2-\frac{3}{2}Z+\frac{1}{2}}\right)-\frac{\frac{1}{2}Z^2}{Z^2-\frac{3}{2}Z+\frac{1}{2}}$$

$$\frac{Y(Z)}{Z}=\frac{\left(2Z^2+\frac{3}{2}Z\right)}{\left(Z-\frac{1}{4}\right)(Z-1)\left(Z-\frac{1}{2}\right)}-\frac{\frac{1}{2}Z}{(Z-1)\left(Z-\frac{1}{2}\right)}=\frac{\left(2Z^2+\frac{3}{2}Z\right)-\frac{1}{2}Z\left(Z-\frac{1}{4}\right)}{\left(Z-\frac{1}{4}\right)(Z-1)\left(Z-\frac{1}{2}\right)}$$

$$=\frac{2Z^2+\frac{3}{2}Z-\frac{1}{2}Z^2+\frac{1}{8}Z}{\left(Z-\frac{1}{4}\right)(Z-1)\left(Z-\frac{1}{2}\right)}=\frac{\frac{3}{2}Z^2+\frac{13}{8}Z}{\left(Z-\frac{1}{4}\right)(Z-1)\left(Z-\frac{1}{2}\right)}$$

$$=\frac{A}{\left(Z-\frac{1}{4}\right)}+\frac{B}{(Z-1)}+\frac{C}{\left(Z-\frac{1}{2}\right)}$$

$$\frac{3}{2}Z^2+\frac{13}{8}Z=A(Z-1)\left(Z-\frac{1}{2}\right)+B\left(Z-\frac{1}{4}\right)\left(Z-\frac{1}{2}\right)+C\left(Z-\frac{1}{4}\right)(Z-1)$$

At $Z=\frac{1}{4}$: $A=\frac{8}{3}$

At $Z=1$:$B=\frac{25}{3}$

At $Z=\frac{1}{2}$:$C=\frac{-19}{2}$

$$\therefore \frac{Y(Z)}{Z}=\frac{\frac{8}{3}}{Z-\frac{1}{4}}+\frac{\frac{25}{3}}{Z-1}-\frac{\frac{19}{2}}{Z-\frac{1}{2}}$$

$$Y(Z)=\frac{8}{3}\frac{Z}{Z-\frac{1}{4}}+\frac{25}{3}\frac{Z}{Z-1}-\frac{19}{2}\frac{Z}{Z-\frac{1}{2}}$$

Applying inverse Z transform

$$y(n) = \frac{8}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{25}{3}u(n) - \frac{19}{2}\left(\frac{1}{2}\right)^n u(n)$$

Q.4: Convolve the following sequences using Tabulation method

$$x(n) = \frac{n}{3}\ for\ 0 \le n \le 6$$

$$h(n) = 1\ for - 2 \le n \le 2$$

$$x(n) = \left\{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}\right\} \qquad h(n) = \left\{1, 1, \underset{\uparrow}{1}, 1, 1\right\}$$

Tabulation method

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad or \quad y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$x(n) = \left\{\underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}\right\} \qquad h(n) = \left\{1, 1, \underset{\uparrow}{1}, 1, 1\right\}$$

k	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
$x(k)$					0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{6}{3}$			
$h(k)$			1	1	1	1	1							
$h(-k)$			1	1	1	1	1							
$h(-k-2)$	1	1	1	1	1									
$h(-k-1)$		1	1	1	1	1								
$h(-k)$			1	1	1	1								
$h(-k+1)$				1	1	1	1	1						
$h(-k+2)$					1	1	1	1	1					
$h(-k+3)$						1	1	1	1	1				
$h(-k+4)$							1	1	1	1	1			
$h(-k+5)$								1	1	1	1			
$h(-k+6)$									1	1	1	1	1	
$h(-k+7)$										1	1	1	1	
$h(-k+8)$											1	1	1	1

$y(-2) = (0)(1) = 0$

$y(-1) = (0)(1) + \left(\frac{1}{3}\right)(1) = \frac{1}{3}$

$y(0) = (0)(1) + \left(\frac{1}{3}\right)(1) + \left(\frac{2}{3}\right)(1) = 1$

$y(1) = (0)(1) + \left(\frac{1}{3}\right)(1) + \left(\frac{2}{3}\right)(1) + \left(\frac{3}{3}\right)(1) = 2$

$y(2) = (0)(1) + \left(\frac{1}{3}\right)(1) + \left(\frac{2}{3}\right)(1) + \left(\frac{3}{3}\right)(1) + \left(\frac{4}{3}\right)(1) = \frac{10}{3}$

$$y(3) = \left(\frac{1}{3}\right)(1) + \left(\frac{2}{3}\right)(1) + \left(\frac{3}{3}\right)(1) + \left(\frac{4}{3}\right)(1) + \left(\frac{5}{3}\right)(1) = 5$$
$$y(4) = \left(\frac{2}{3}\right)(1) + \left(\frac{3}{3}\right)(1) + \left(\frac{4}{3}\right)(1) + \left(\frac{5}{3}\right)(1) + \left(\frac{6}{3}\right)(1) = \frac{20}{3}$$
$$y(5) = \left(\frac{3}{3}\right)(1) + \left(\frac{4}{3}\right)(1) + \left(\frac{5}{3}\right)(1) + \left(\frac{6}{3}\right)(1) = 6$$
$$y(6) = \left(\frac{4}{3}\right)(1) + \left(\frac{5}{3}\right)(1) + \left(\frac{6}{3}\right)(1) = 5$$
$$y(7) = \left(\frac{5}{3}\right)(1) + \left(\frac{6}{3}\right)(1) = \frac{11}{3}$$
$$y(8) = \left(\frac{6}{3}\right)(1) = 2$$
$$\therefore y(n) = \left\{0,\ \frac{1}{3},\ \underset{\uparrow}{1},\ 2,\ \frac{10}{3},\ 5,\ \frac{20}{3},\ 6,\ 5,\ \frac{11}{3},\ 2\right\}$$

Q.5: Obtain Cascade form realization

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

Solution:

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

Taking Z-transform

$$Y(Z) - \frac{1}{4}Z^{-1}Y(Z) - \frac{1}{8}Z^{-2}Y(Z) = X(Z) + 3Z^{-1}X(Z) + 2Z^{-1}X(Z)$$
$$\frac{Y(Z)}{X(Z)} = \frac{1 + 3Z^{-1} + 2Z^{-1}}{1 - \frac{1}{4}Z^{-1} - \frac{1}{8}Z^{-2}} = \frac{(1 + Z^{-1})(1 + 2Z^{-1})}{(1 - \frac{1}{2}Z^{-1})(1 + \frac{1}{4}Z^{-1})} = H_1(Z).H_2(Z)$$

$$H_1(Z) = \frac{(1 + Z^{-1})}{(1 - \frac{1}{2}Z^{-1})} \qquad H_2(Z) = \frac{(1 + 2Z^{-1})}{(1 + \frac{1}{4}Z^{-1})}$$

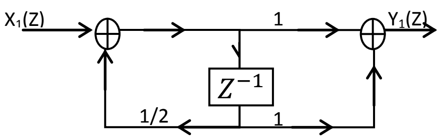


Fig 5.19 Direct form II structure of $H_1(Z)$

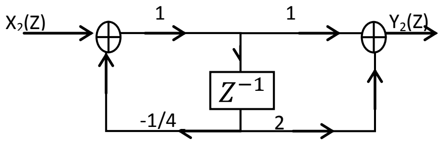


Fig 5.20 Direct form II structure of $H_2(Z)$

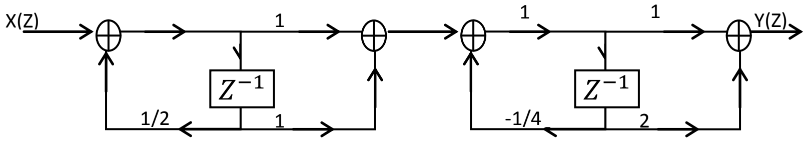


Fig 5.21 Cascade form

Q.6: Obtain Parallel form realization

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) + 3x(n-1) + 2x(n-2)$$

Solution:

$$y(n)-\frac{1}{4}y(n-1)-\frac{1}{8}y(n-2)=x(n)+3x(n-1)+2x(n-2)$$
$$\frac{Y(Z)}{X(Z)}=\frac{1+3Z^{-1}+2Z^{-2}}{1-\frac{1}{4}Z^{-1}-\frac{1}{8}Z^{-2}}$$

$$\begin{array}{r|l} & -16 \\ \hline -\frac{1}{8}Z^{-2}-\frac{1}{4}Z^{-1}+1 & 2Z^{-2}+3Z^{-1}+1 \\ & 2Z^{-2}+4Z^{-1}-16 \\ & (-)\quad(-)\qquad(+) \\ \hline & -Z^{-1}+17 \end{array}$$

$$\frac{Y(Z)}{X(Z)}=-16+\frac{-Z^{-1}+17}{1-\frac{1}{4}Z^{-1}-\frac{1}{8}Z^{-2}}=-16+\frac{17-Z^{-1}}{(1-\frac{1}{2}Z^{-1})(1+\frac{1}{4}Z^{-1})}$$
$$Let\quad \frac{17-Z^{-1}}{(1-\frac{1}{2}Z^{-1})(1+\frac{1}{4}Z^{-1})}=\frac{A}{1-\frac{1}{2}Z^{-1}}+\frac{B}{1+\frac{1}{4}Z^{-1}}$$
$$17-Z^{-1}=A\left(1+\frac{1}{4}Z^{-1}\right)+B(1-\frac{1}{2}Z^{-1})$$

$at\ Z^{-1}=-4$

$$17+4=B(1-\frac{1}{2}(-4))$$
$$\therefore\ B=7$$

$at\ Z^{-1}=2$

$$17-2=A(1+\frac{1}{4}(2))$$
$$\therefore A=10$$

$$\therefore H(Z)=-16+\frac{10}{1-\frac{1}{2}Z^{-1}}+\frac{7}{1+\frac{1}{4}Z^{-1}}$$

$$H_1(Z)=\frac{Y_1(Z)}{X_1(Z)}=\frac{10}{1-\frac{1}{2}Z^{-1}}$$
$$\frac{Y_1(Z)}{W_1(Z)}=10\Rightarrow Y_1(Z)=10W_1(Z)$$
$$\frac{W_1(Z)}{X_1(Z)}=\frac{1}{1-\frac{1}{2}Z^{-1}}$$
$$W_1(Z)=X_1(Z)+\frac{1}{2}Z^{-1}W_1(Z)$$

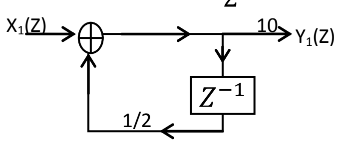


Fig 5.22 Direct form II structure of $H_1(Z)$

$$H_2(Z)=\frac{Y_2(Z)}{X_2(Z)}=\frac{7}{1+\frac{1}{4}Z^{-1}}$$
$$\frac{Y_2(Z)}{W_2(Z)}=7\Rightarrow Y_2(Z)=7W_2(Z)$$
$$\frac{W_2(Z)}{X_2(Z)}=\frac{1}{1+\frac{1}{4}Z^{-1}}$$
$$W_2(Z)=X_2(Z)-\frac{1}{4}Z^{-1}W_2(Z)$$

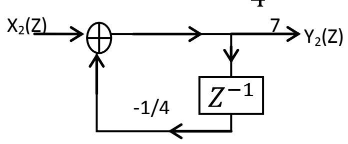


Fig 5.23 Direct form II structure of $H_2(Z)$

Combining figures Fig 5.22 and Fig 5.23 we can form parallel form realization as shown in

Fig 5.24

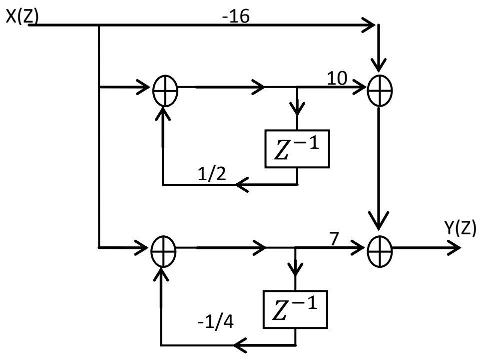
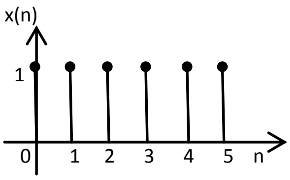


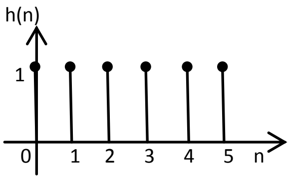
Fig 5.24 Parallel form

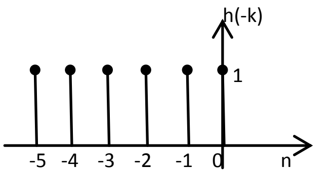
Q.7: Convolve the following discrete time signals using graphical convolution $x(n) = h(n) = u(n)$

Solution:

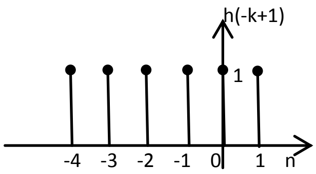
$$x(n) = u(n) = 1; n \geq 0$$
$$h(n) = u(n) = 1; n \geq 0$$



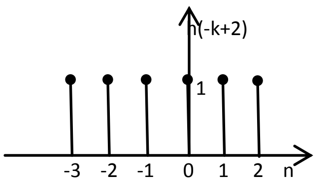




$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$
$$when\ n = 0$$
$$y(0) = (1)(1) = 1$$
$$when\ n = 1$$
$$y(1) = (1)(1) + (1)(1) = 2$$



$$when\ n = 2$$
$$y(2) = (1)(1) + (1)(1) + (1)(1) = 3$$



$$\therefore y(n) = \{1,2,3,4,5, \ldots .\}$$

Q.8: Compute linear convolution.

$$x(n) = \{2,2,0,1,1\} \quad h(n) = \{1,2,3,4\}$$

Solution:

	2	2	0	1	1
1	2	2	0	1	1
2	4	4	0	2	2
3	6	6	0	3	3
4	8	8	0	4	4

$$y(n) = x(n) * h(n) = \{2,6,10,15,11,5,7,4\}$$